

# What Does Probability have to do with God?

Exploring Bayesian Reasoning in Theological Problems

Dr. Brian Blais  
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<https://bblais.github.io>



# Who am I

- Dr Brian Blais is a professor of Science in the School for Health and Behavioral Sciences, Bryant University, Rhode Island. He has a PhD in Physics from Brown University and for many years was a Visiting Professor in the Institute for Brain and Neural Systems, Brown University. His focus has been on computational and statistical methods applied to a wide range of fields such as the neuroscience of vision, paleoclimate, disease modeling, and most recently the textual properties of the New Testament. He has a personal interest in science education and maintains a blog at <https://bblais.github.io> where he explores the intersection of science and society, often dealing with issues in religious thought and pseudoscience.

# Summary

- In this talk, I will present a framework for rational inquiry based on probability theory. Probability theory is a branch of mathematics that deals with uncertainty and how to reason about it. I will explain the basic concepts and principles of probability, and how they can be applied to any domain of interest, including theology, the study of the nature of God and religious beliefs. I will explore some of the key theological concepts, such as belief, faith, miracles, and the existence of God, and how they can be analyzed using probability. I will also compare and contrast the scientific method with other ways of acquiring knowledge, such as revelation and intuition. My aim is to provide a clear and consistent way of thinking about these topics, and to reveal the hidden assumptions and implications of various theological arguments. Along the way, I will demonstrate some surprising and counterintuitive results that arise from probability theory, and how they can lead to errors in reasoning.

# My Religious Story

- Brought up Catholic
- Left the Church in High School
- Became a Deist in Early College
- Became an Atheist in Late College
- Always a strong proponent of scientific thinking and education
- Enjoy exploring (criticizing) pseudoscience in all its forms as a means of education — UFOs, magnetic therapy, astrology, faith healings, miracles, etc...

# Outline

- Intro to probability notation
- A simple example to show some methods
- Bayesian calculation of the probability for the Resurrection of Jesus
- God's existence
  - Lack of imagination
  - Simplicity
- Evidence, Testimony, and Miracles
- Faith and Trust
- Priors vs Likelihoods

# E. T. Jaynes and Probability

- (I) Degrees of plausibility are represented by real numbers
- (II) Qualitative correspondence with common sense
  - (a) direction of values is correct
  - (b) consistent with true/false logic (aka Boolean logic)
- (IIIa) If a conclusion can be reasoned out in more than one way, then every possible way must lead to the same result.
- (IIIb) Always takes into account all of the evidence
- (IIIc) Equivalent states of knowledge have equivalent plausibility assignments

Probability Theory  
The Logic of Science

E. T. JAYNES

CAMBRIDGE

# Rules for Plausibility

- Convention:

- $p(A) = 0$  certain that A is false
- $p(A) = 1$  certain that A is true

- Limited Sum Rule

$$p(A) + p(\bar{A}) = 1$$

- Full Sum Rule (“or”)

$$p(A + B) = p(A) + p(B) - p(AB)$$

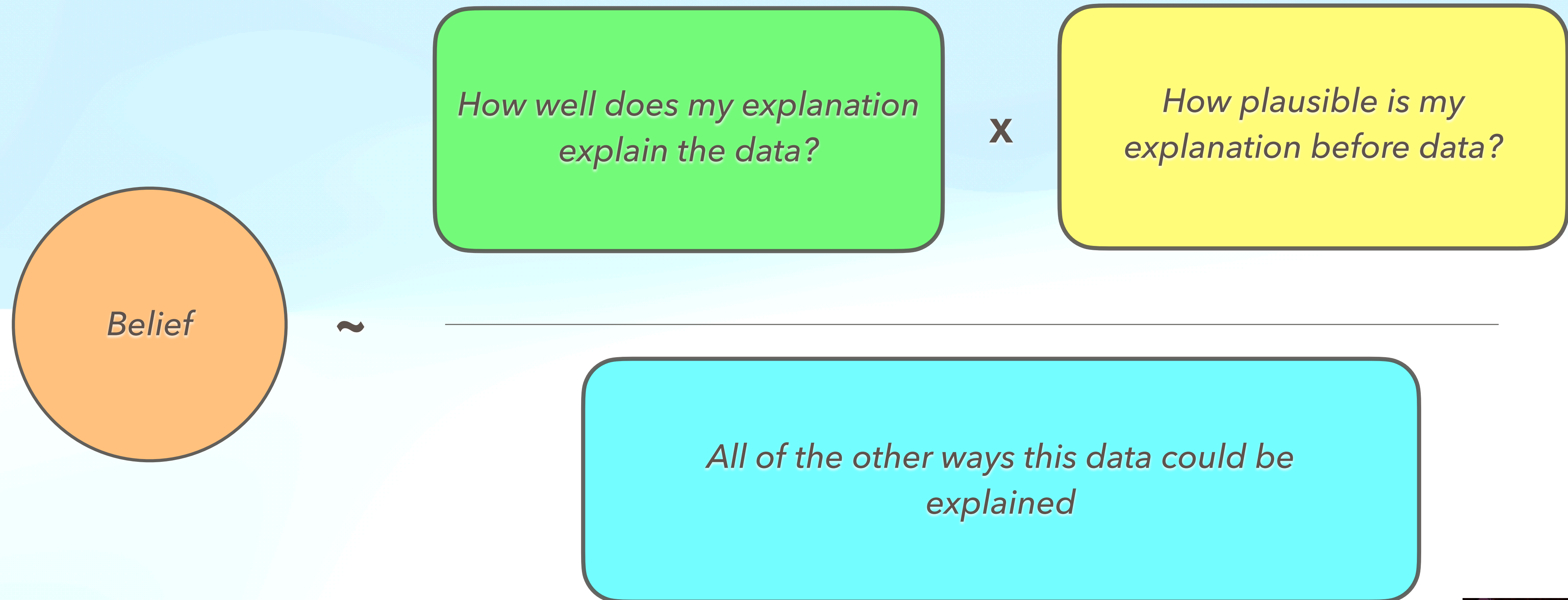
- Product Rule (“and”)

$$\begin{aligned} p(AB) &= p(A|B)p(B) \\ &= p(B|A)p(A) \end{aligned}$$

- Bayes Rule

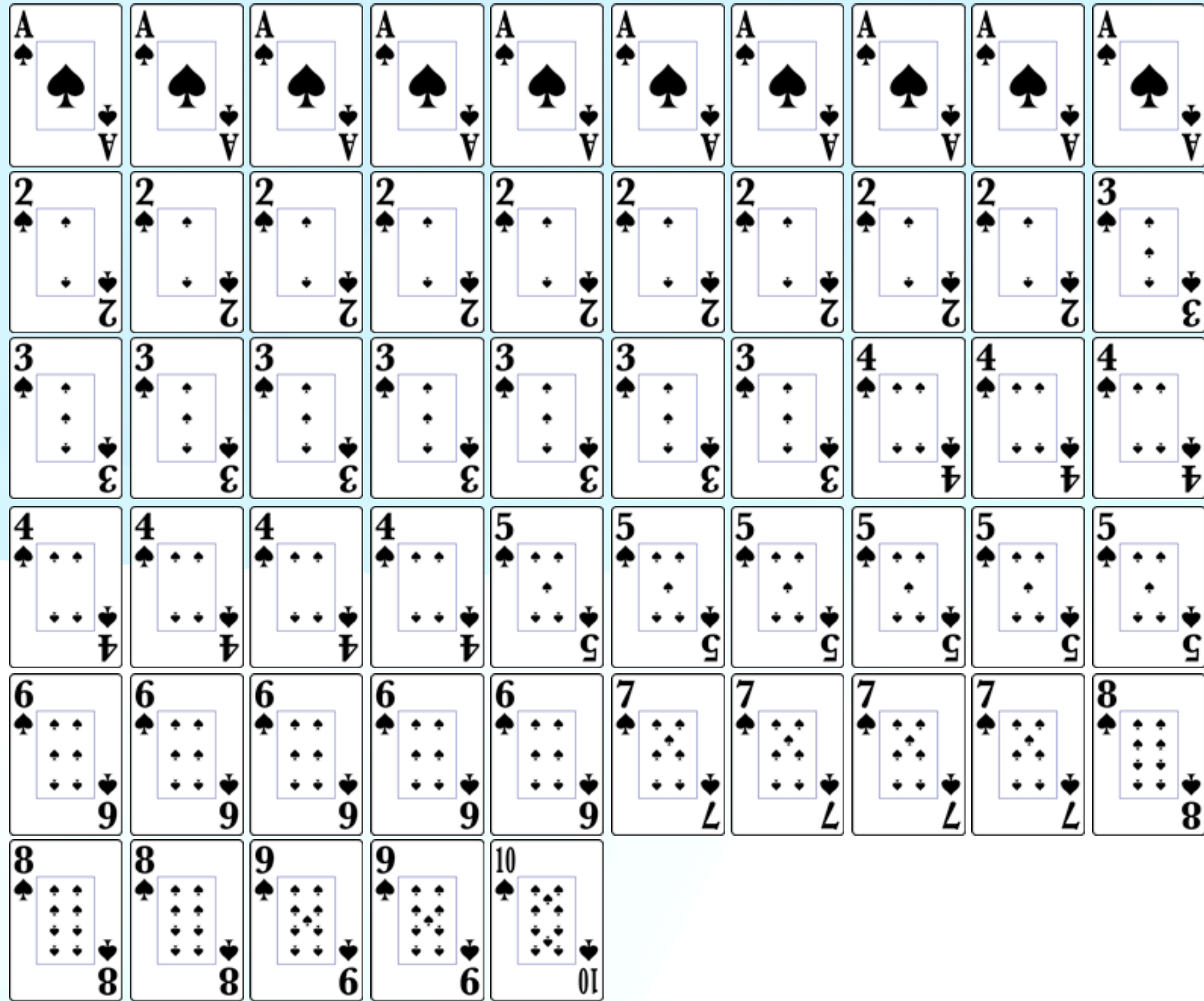
$$\underbrace{p(A|B)}_{\text{posterior}} = \frac{\overbrace{p(B|A)}^{\text{likelihood}} \overbrace{p(A)}^{\text{prior}}}{\underbrace{p(B)}_{\text{normalization}}}$$

# Bayes Theorem without Math



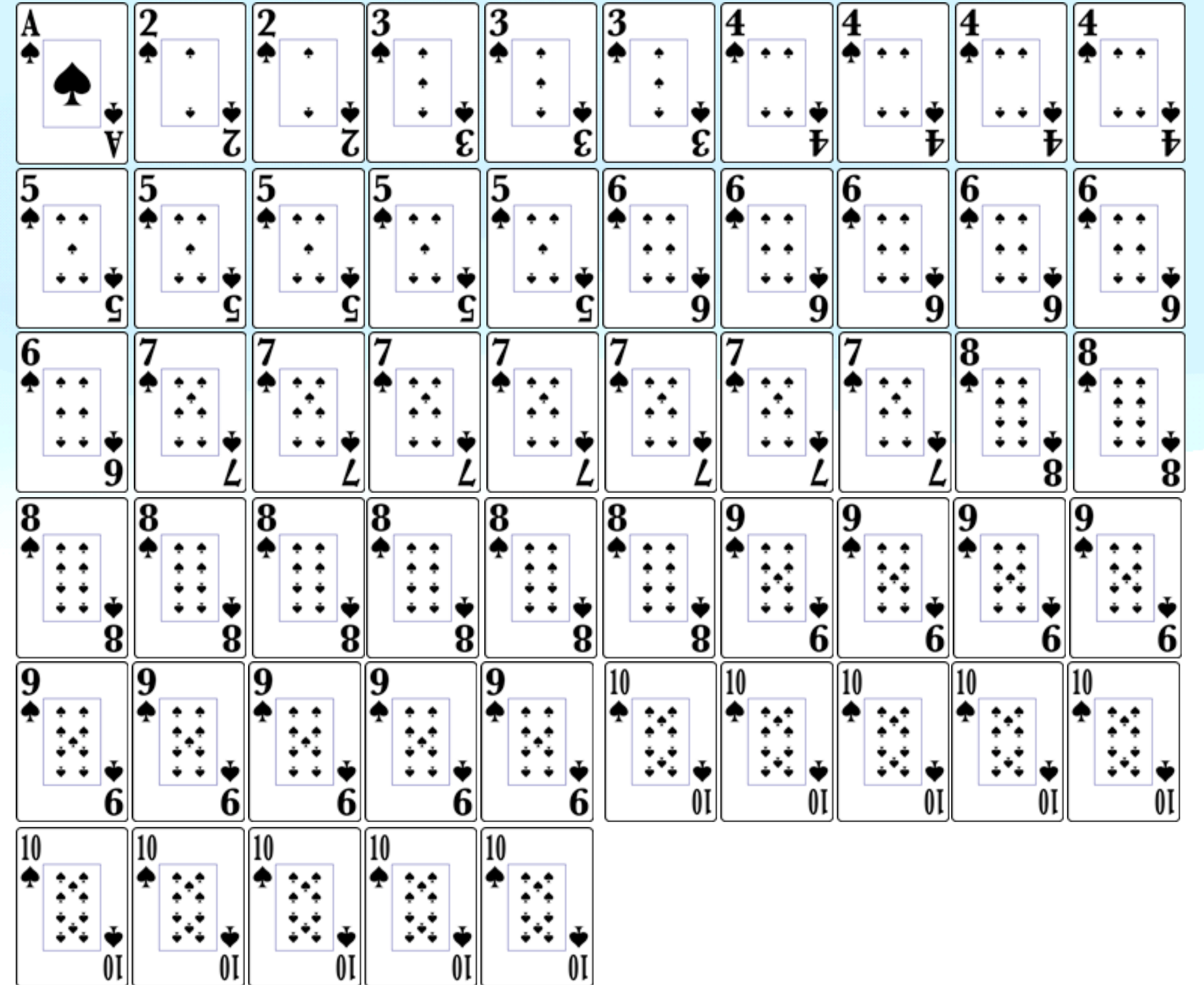


# Low Deck



$$N = 55$$

# High Deck

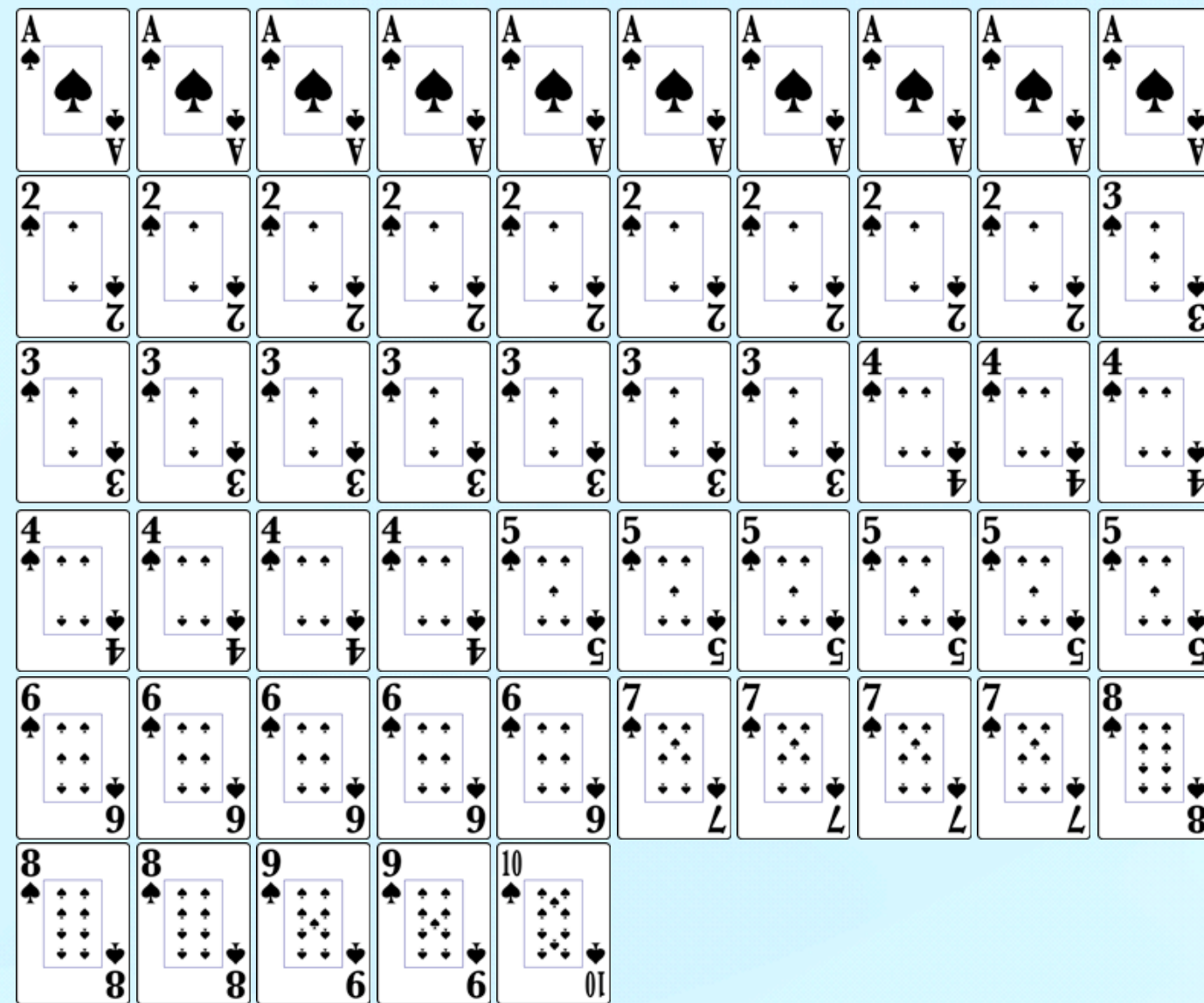


$$N = 55$$



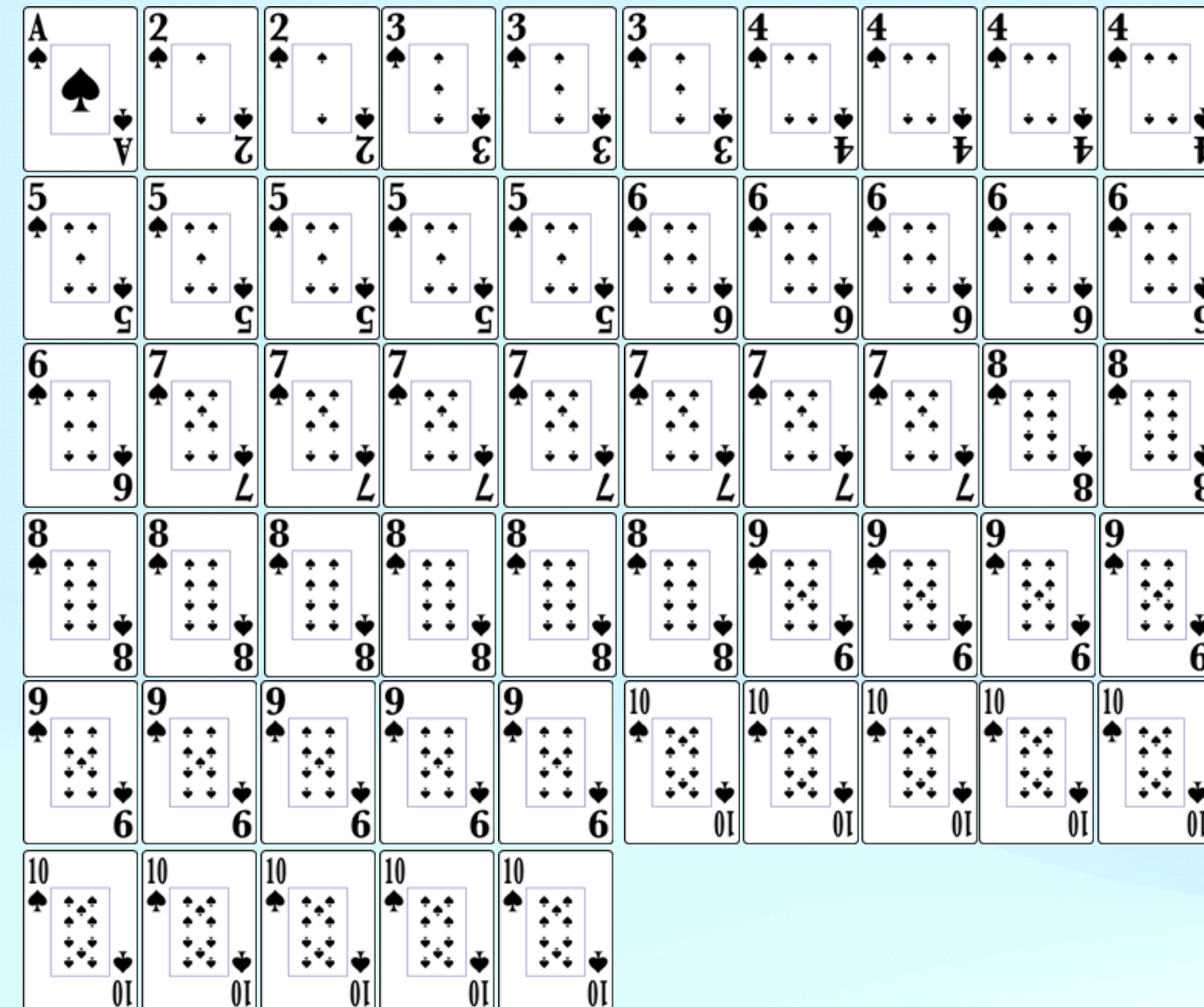


# Low Deck



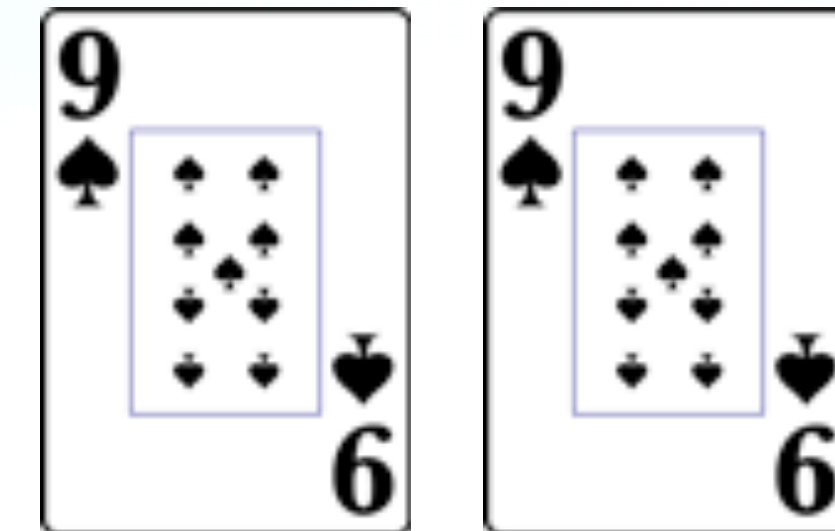
$N = 55$

# High Deck



$N = 55$

- You're given an unknown deck, told it's either the High or Low deck
- Take draws from the deck to determine which deck you're likely holding
- After each draw you reinsert the card into the deck and reshuffle (math convenience)
- Data:



Intuition?



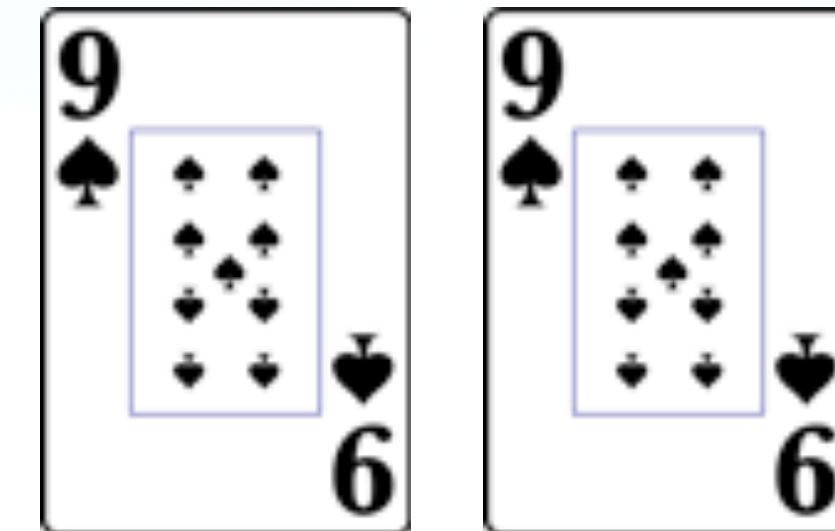
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## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- You're given an unknown deck, told it's either the High or Low deck
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- Data:



Intuition?



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- You're given an unknown deck, told it's either the High or Low deck
- Take draws from the deck to determine which deck you're likely holding
- After each draw you reinsert the card into the deck and reshuffle (math convenience)

- 
- The decks you're comparing
  - The draws of cards
  - Goal:

= models of the world  
= data  
the most likely model given the data

## Low Deck

10 A's, 9 2's, 8 3's, ..., 2 9's, 1 10 = 55 cards

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- You're given an unknown deck, told it's either the High or Low deck
- Take draws from the deck to determine which deck you're likely holding
- After each draw you reinsert the card into the deck and reshuffle (math convenience)

- 
- The decks you're comparing = models of the world
  - The draws of cards = data
  - Goal: the most likely model given the data

- Math:

$$P(H | \text{data}) \sim P(\text{data} | H)P(H)$$

$$P(L | \text{data}) \sim P(\text{data} | L)P(L)$$

Note: Doing the Calculation in Two Steps

<https://bblais.github.io/posts/2019/Jan/14/stats-for-everyone/>



## Low Deck

10 A's, 9 2's, 8 3's, ..., 2 9's, 1 10 = 55 cards

## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- Either the High or Low deck
- Data: draws from the deck
- Reinsert and reshuffle

- 
- The decks you're comparing = models of the world
  - The draws of cards = data
  - Goal: the most likely model given the data

- Math:

$$P(H | \text{data}) \sim P(\text{data} | H)P(H)$$

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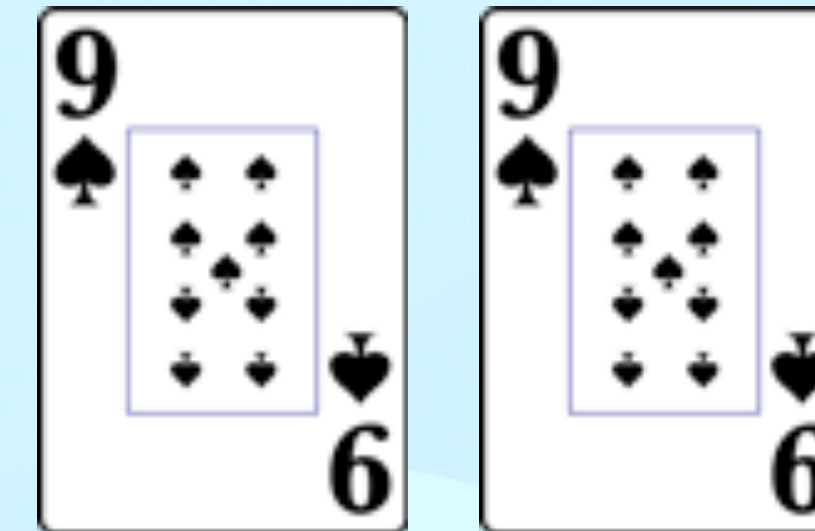
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## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- Either the High or Low deck
- Reinsert and reshuffle
- Data:



- 
- models of the world = holding the High deck ( $H$ ) or the Low deck ( $L$ )
  - Data = draw a 9, reinsert and reshuffle, draw another 9
  - Goal: the most likely model given the data

- Math:

$$P(H | 9,9) \sim P(9,9 | H)P(H)$$

$$P(L | 9,9) \sim P(9,9 | L)P(L)$$

Note: Doing the  
Calculation in  
Two Steps

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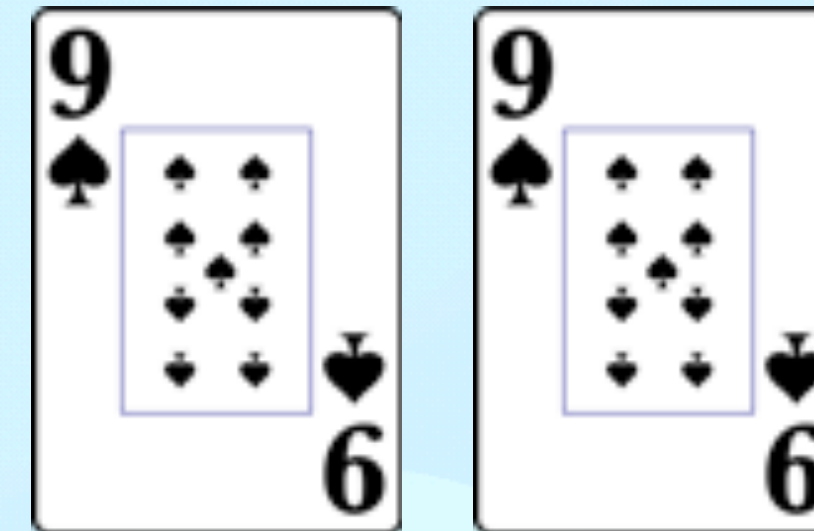
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## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- Either the High or Low deck
- Reinsert and reshuffle
- Data:



- 
- models of the world = holding the High deck ( $H$ ) or the Low deck ( $L$ )
  - Data = draw a 9, reinsert and reshuffle, draw another 9
  - Goal: the most likely model given the data

• Math:  $P(H | 9,9) \sim P(9,9 | H)P(H) = 9/55 \times 9/55 \times 1/2 = 0.0133$

$$P(L | 9,9) \sim P(9,9 | L)P(L) = 2/55 \times 2/55 \times 1/2 = 0.0006$$

Note: Doing the  
Calculation in  
Two Steps

<https://bblais.github.io/posts/2019/Jan/14/stats-for-everyone/>

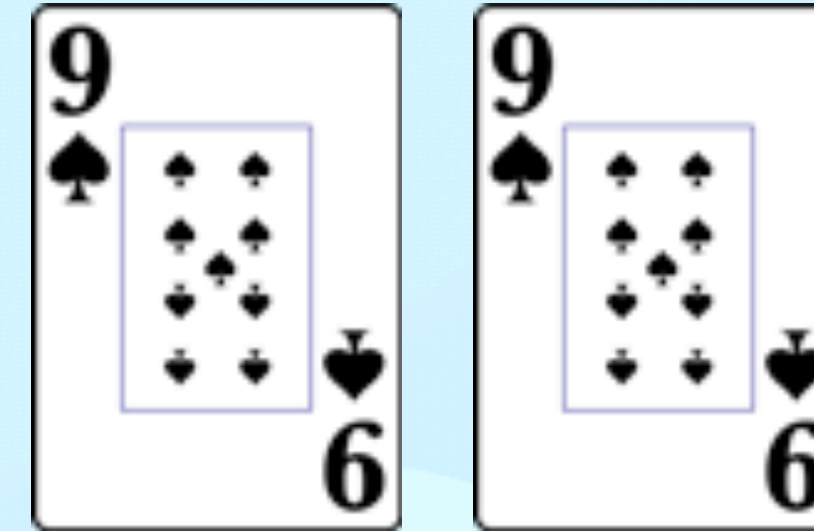
## Low Deck

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## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- Either the High or Low deck
- Reinsert and reshuffle
- Data:



- 
- models of the world = holding the High deck ( $H$ ) or the Low deck ( $L$ )
  - Data = draw a 9, reinsert and reshuffle, draw another 9
  - Goal: the most likely model given the data

• Math:  $P(H | 9,9) \sim P(9,9 | H)P(H) = 9/55 \times 9/55 \times 1/2 = 0.0133$

$P(L | 9,9) \sim P(9,9 | L)P(L) = 2/55 \times 2/55 \times 1/2 = 0.0006$

-----  
 $T = 0.0139$

Note: Doing the  
 Calculation in  
 Two Steps

<https://bblais.github.io/posts/2019/Jan/14/stats-for-everyone/>

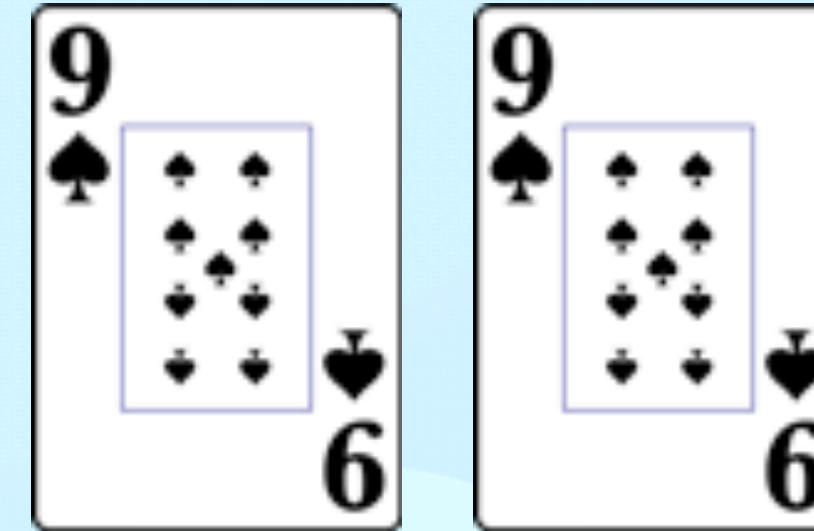
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## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- Either the High or Low deck
- Reinsert and reshuffle
- Data:



- 
- models of the world = holding the High deck ( $H$ ) or the Low deck ( $L$ )
  - Data = draw a 9, reinsert and reshuffle, draw another 9
  - Goal: the most likely model given the data

• Math:  $P(H | 9,9) \sim P(9,9 | H)P(H) = 9/55 \times 9/55 \times 1/2 = 0.0133/T = 0.963$

$P(L | 9,9) \sim P(9,9 | L)P(L) = 2/55 \times 2/55 \times 1/2 = 0.0006/T = 0.047$

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$T = 0.0139$

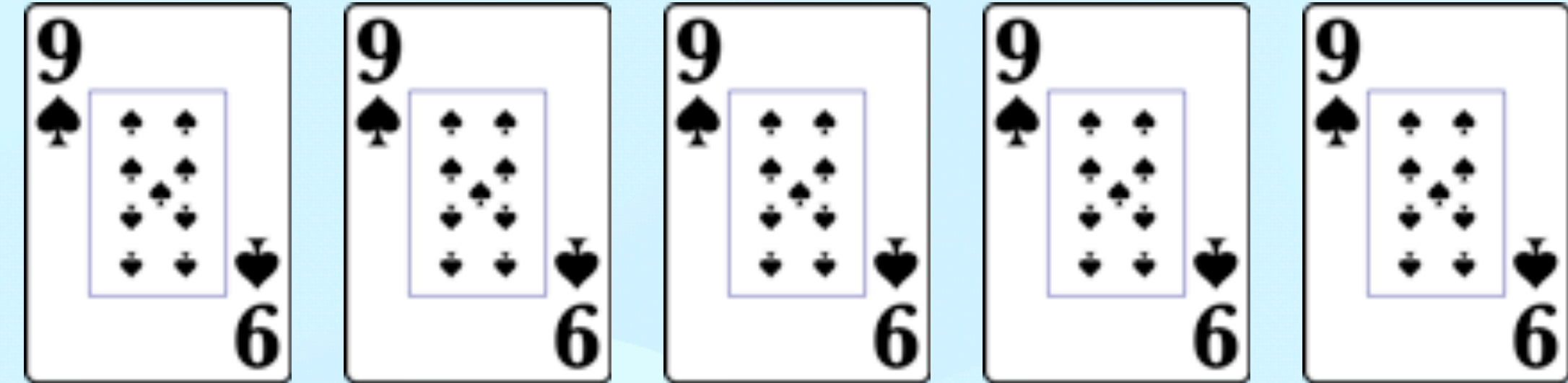
## Low Deck

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## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- Either the High or Low deck
- Reinsert and reshuffle
- Data: 5 9's in a row

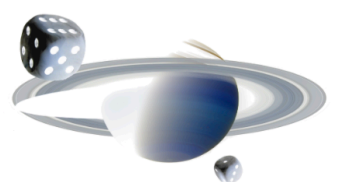


- 
- models of the world = holding the High deck ( $H$ ) or the Low deck ( $L$ )
  - Data = draw a 5 9's in a row, with reinsert and reshuffle
  - Goal: the most likely model given the data

• Math:  $P(H | 5 \text{ 9's}) \sim P(5 \text{ 9's} | H)P(H) = (9/55)^5 \times 1/2 = 5.8663 \cdot 10^{-5}/T = 0.9995$

$$P(L | 5 \text{ 9's}) \sim P(5 \text{ 9's} | L)P(L) = (2/55)^5 \times 1/2 = 3.2 \cdot 10^{-8}/T = 0.0005$$

$$T = 5.8663 \cdot 10^{-5} + 3.2 \cdot 10^{-8} = 5.8695 \cdot 10^{-5}$$



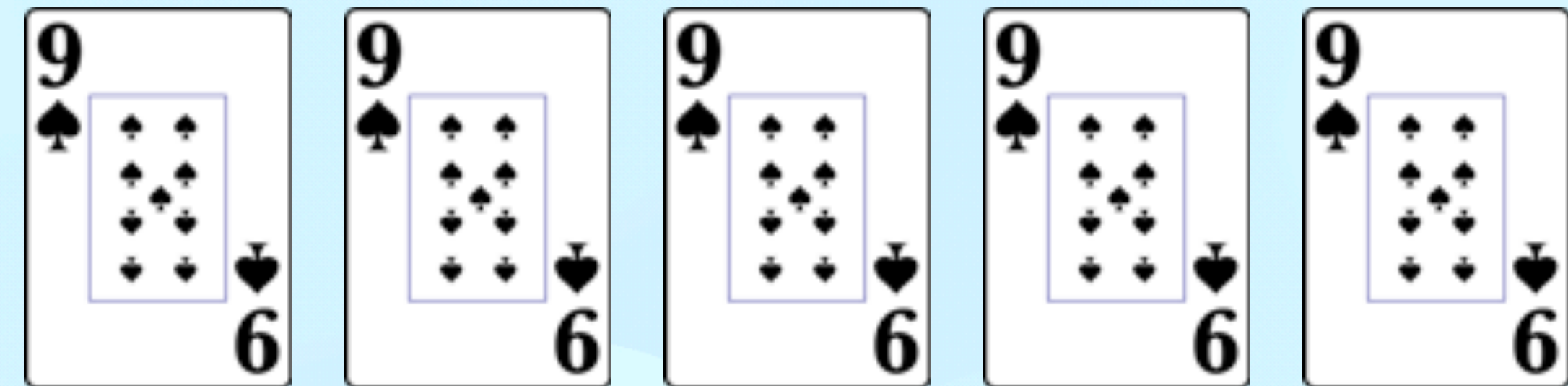
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## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- Either the High or Low deck
- Reinsert and reshuffle
- Data: 5 9's in a row



- 
- models of the world = holding the High deck ( $H$ ) or the Low deck ( $L$ )
  - Data = draw a 5 9's in a row, with reinsert and reshuffle
  - Goal: the most likely model given the data

- Math:  $P(H | 5 \text{ 9's}) \sim P(5 \text{ 9's} | H)P(H) = 0.9995$

$$P(L | 5 \text{ 9's}) \sim P(5 \text{ 9's} | L)P(L) = 0.0005$$

Any Issue with this statement?  
Drawing 5 9's in a row in the process is much more likely on the High deck than on the Low Deck



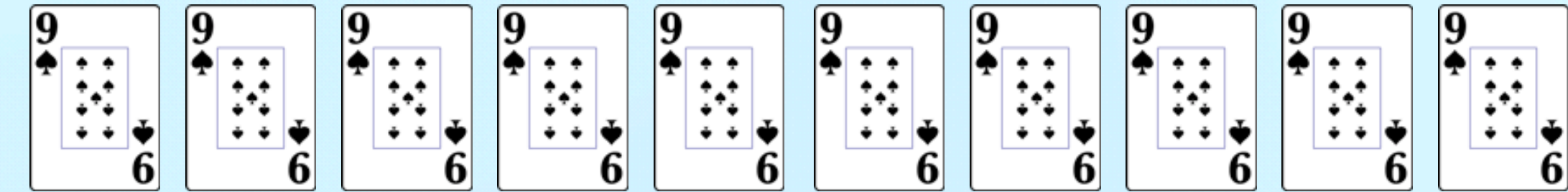
## Low Deck

10 A's, 9 2's, 8 3's, ..., 2 9's, 1 10 = 55 cards

## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

- Either the High or Low deck
- Reinsert and reshuffle
- Data: 10 9's in a row



- 
- models of the world = holding the High deck ( $H$ ) or the Low deck ( $L$ )
  - Data = draw a 10 9's in a row, with reinsert and reshuffle
  - Goal: the most likely model given the data

- Math:  $P(H | 10 \text{ 9's}) \sim P(10 \text{ 9's} | H)P(H) = 0.99999997$

$$P(L | 10 \text{ 9's}) \sim P(10 \text{ 9's} | L)P(L) = 3 \cdot 10^{-7}$$

Any Issue with this statement?  
Drawing 10 9's in a row in the process is much more likely on the High deck than on the Low Deck



Told that → Either the High or Low deck

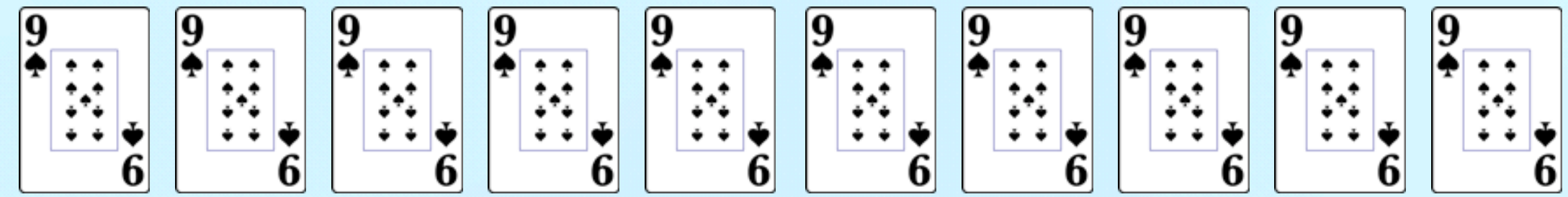
### Low Deck

10 A's, 9 2's, 8 3's, ..., 2 9's, 1 10 = 55 cards

- Reinsert and reshuffle
- Data: 10 9's in a row

### High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards



- models of the world = holding the High deck ( $H$ ) or the Low deck ( $L$ )
- Data = draw a 10 9's in a row, with reinsert and reshuffle
- Goal: the most likely model given the data

- Math:  $P(H | 10\ 9's) \sim P(10\ 9's | H)P(H) = 0.99999997$
- $P(L | 10\ 9's) \sim P(10\ 9's | L)P(L) = 3 \cdot 10^{-7}$

Any Issue with this statement?  
Drawing 10 9's in a row in the process is much more likely on the High deck than on the Low Deck



## Low Deck

10 A's, 9 2's, 8 3's, ..., 2 9's, 1 10 = 55 cards

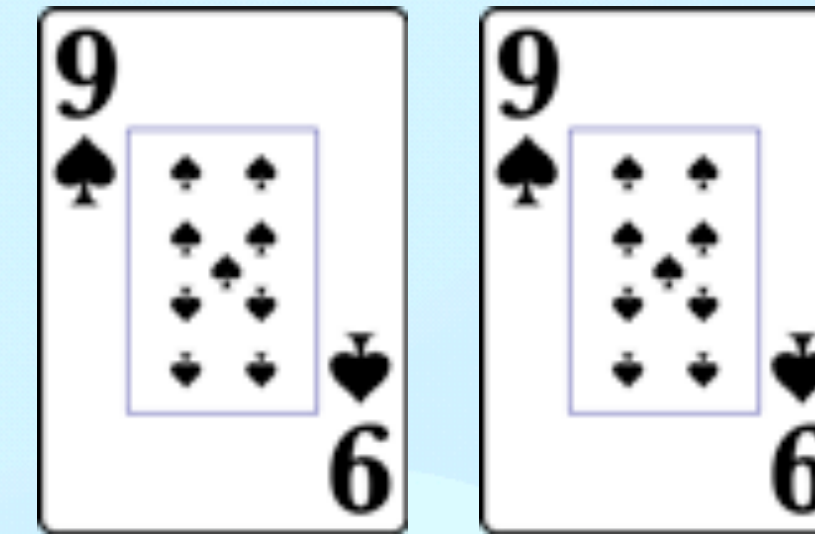
## High Deck

1 A, 2 2's, 3 3's, ..., 9 9's, 10 10's = 55 cards

## 9's Deck

0 A, 0 2's, 0 3's, ..., 55 9's, 0 10's = 55 cards

- Either the High, Low, or Nines deck
- Reinsert and reshuffle
- Data: 2 9's in a row



- models of the world = High ( $H$ ), Low ( $L$ ), or Nines deck ( $N$ )
- Data = draw a 2 9's in a row, with reinsert and reshuffle
- Goal: the most likely model given the data

## • Math:

$$P(H | 9,9) \sim P(9,9 | H)P(H) = 9/55 \times 9/55 \times 0.499 = 0.01333/T = 0.834$$

$$P(L | 9,9) \sim P(9,9 | L)P(L) = 2/55 \times 2/55 \times 0.499 = 0.0006/T = 0.041$$

$$P(N | 9,9) \sim P(9,9 | N)P(N) = 55/55 \times 55/55 \times 0.002 = 0.0020/T = 0.125$$

$$T = 0.01602$$



- Priors

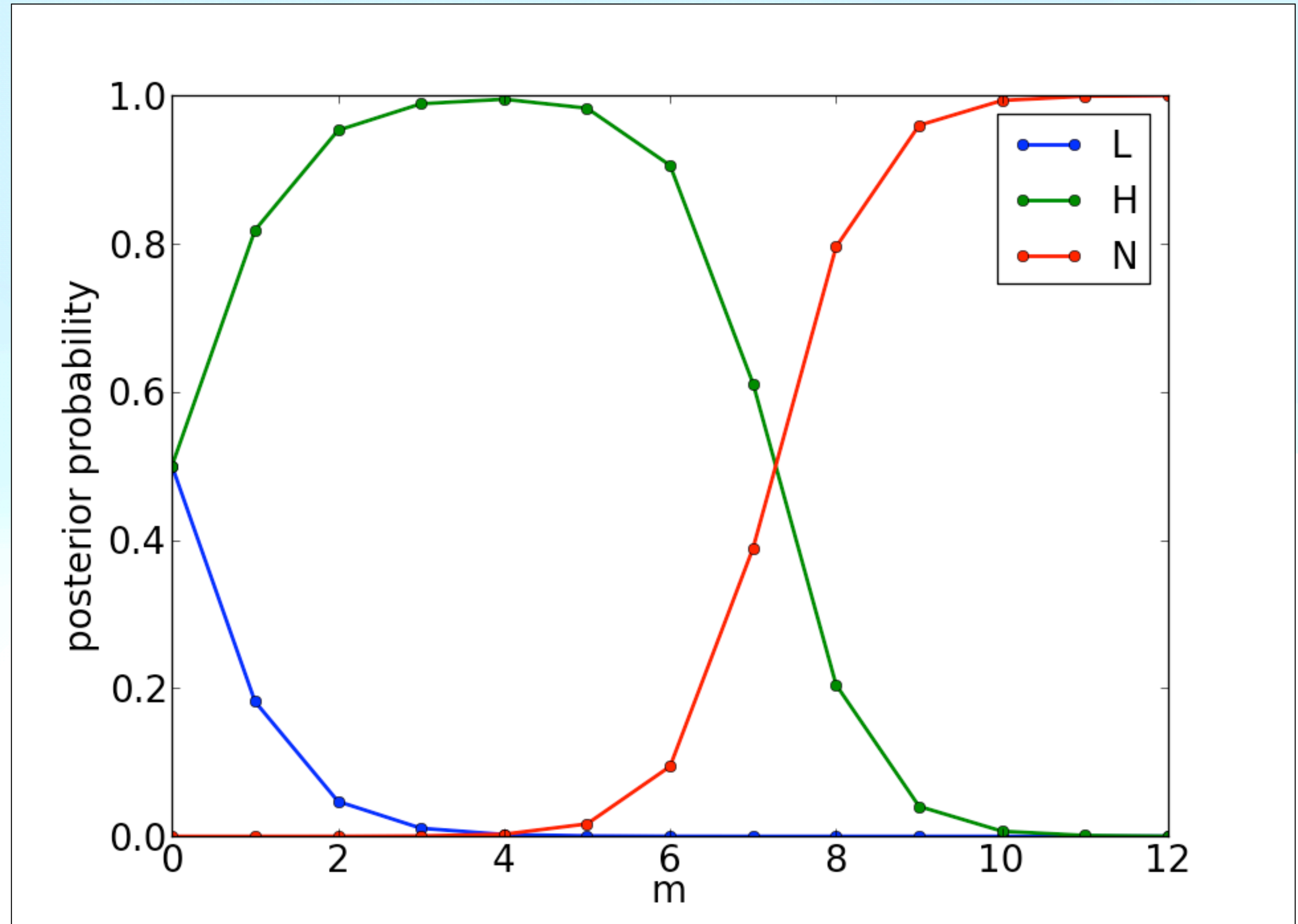
$$P(H) = 0.49999995$$

$$P(L) = 0.49999995$$

$$P(N) = 1 \cdot 10^{-6}$$

- Data:

- Draw  $m$  9's in a row



•Priors

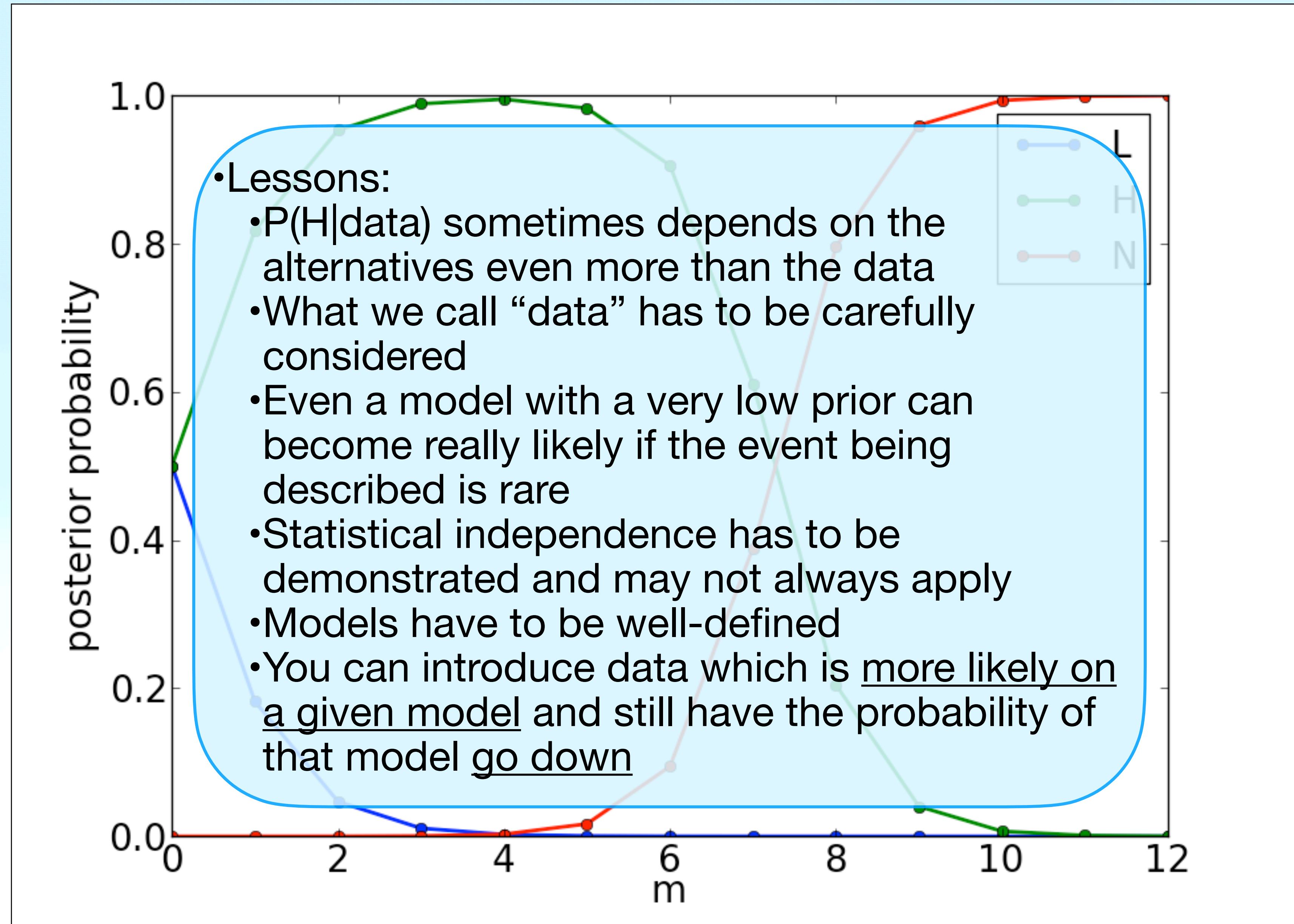
$$P(H) = 0.4999995$$

$$P(L) = 0.4999995$$

$$P(N) = 1 \cdot 10^{-6}$$

•Data:

- Draw  $m$  9's in a row



- Models of the world:
  - Resurrection ( $R$ )
  - Not-Resurrection ( $\neg R$ )
- Data:
  - Women found the tomb empty ( $W$ )
  - 13 disciples saw Jesus after death ( $D_{13}$ )
  - Saul/Paul was converted ( $S$ )

## The Argument from Miracles: A Cumulative Case for the Resurrection of Jesus of Nazareth

TIMOTHY MCGREW AND LYDIA MCGREW

$$P(R | W, D_{13}, S) \sim P(W, D_{13}, S | R)P(R)$$
$$P(\neg R | W, D_{13}, S) \sim P(W, D_{13}, S | \neg R)P(\neg R)$$

McGrew, T., & McGrew, L. (2009). The argument from miracles: a cumulative case for the resurrection of Jesus of Nazareth. *The Blackwell companion to natural theology*, 593-662.

- Models of the world:
  - Resurrection ( $R$ )
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## The Argument from Miracles: A Cumulative Case for the Resurrection of Jesus of Nazareth

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$$P(R | W, D_{13}, S) \sim P(W, D_{13}, S | R)P(R)$$

$$P(\neg R | W, D_{13}, S) \sim P(W, D_{13}, S | \neg R)P(\neg R)$$

$$\frac{P(W | R)}{P(W | \neg R)} \cdot \left( \frac{P(D_1 | R)}{P(D_1 | \neg R)} \right)^{13} \cdot \frac{P(S | R)}{P(S | \neg R)} = \frac{100}{1} \cdot \left( \frac{1000}{1} \right)^{13} \cdot \frac{1000}{1} = 10^{44}$$

- Assumptions:
  - Observations from the disciples were independent
  - Single hallucination is 1000:1 against
- Conclusion:
  - Resurrection hypothesis much more likely

McGrew, T., & McGrew, L. (2009). The argument from miracles: a cumulative case for the resurrection of Jesus of Nazareth. *The Blackwell companion to natural theology*, 593-662.

- Models of the world:
  - Resurrection ( $R$ )
  - Not-Resurrection ( $\neg R$ )

Models not well-defined

Lack of imagination for alternatives

- Data:
  - Women found the tomb empty ( $W$ )
  - 13 disciples saw Jesus after death ( $D_{13}$ )
  - Saul/Paul was converted ( $S$ )

Data: We have texts with stories that include...

The Argument from Miracles:  
A Cumulative Case for  
the Resurrection of Jesus  
of Nazareth  
TIMOTHY MCGREW AND LYDIA MCGREW

$$P(R | W, D_{13}, S) \sim P(W, D_{13}, S | R)P(R) \quad \text{Priors ignored}$$

$$P(\neg R | W, D_{13}, S) \sim P(W, D_{13}, S | \neg R)P(\neg R)$$

$$\frac{P(W | R)}{P(W | \neg R)} \cdot \left( \frac{P(D_1 | R)}{P(D_1 | \neg R)} \right)^{13} \cdot \frac{P(S | R)}{P(S | \neg R)} = \frac{100}{1} \cdot \left( \frac{1000}{1} \right)^{13} \cdot \frac{1000}{1} = 10^{44}$$

- Assumptions:
  - Observations from the disciples were independent
  - Single hallucination is 1000:1 against

Independence not justified

- Conclusion:
  - Resurrection hypothesis much more likely

McGrew, T., & McGrew, L. (2009). The argument from miracles: a cumulative case for the resurrection of Jesus of Nazareth. *The Blackwell companion to natural theology*, 593-662.

# An Improvement

- Models of the world:
  - Resurrection by Yahweh ( $R$ ) (still may not be well-defined)
  - Entirely manufactured ( $M$ )
  - Resurrection story incited by visions from early apostles and embellished ( $V$ )
  - James Fodor's RHBS Model ( $F$ ) (reburial, hallucination, cognitive bias, socialization)
  - [...] others
- Data:
  - Texts that we have
  - Knowledge of human psychology, eyewitness testimony limitations, scientific understanding of the universe, ...

Fodor, J. (2022).  
Unreasonable Faith:  
How William Lane Craig  
Overstates the Case for  
Christianity. Ockham  
Publishing Group.

$$P(R | \text{data}) \sim P(\text{data} | R)P(R)$$

$$P(M | \text{data}) \sim P(\text{data} | M)P(M)$$

$$P(V | \text{data}) \sim P(\text{data} | V)P(V)$$

$$P(F | \text{data}) \sim P(\text{data} | F)P(F)$$

[...]



# The Argument from Miracles: A Cumulative Case for the Resurrection of Jesus of Nazareth

TIMOTHY MCGREW AND LYDIA MCGREW

- Bad Apologetics Ep 18 - Bayes Machine goes BRRRRRRRRRR on Digital Gnosis YouTube (<https://www.youtube.com/watch?v=yeCBpO7pSRM>) 9 hours!

Or a text summary at

- <https://bblais.github.io/posts/2021/Aug/29/bad-apologetics-ep-18-bayes-machine-goes-brrrrrrrrr/>

McGrew, T., & McGrew, L. (2009). The argument from miracles: a cumulative case for the resurrection of Jesus of Nazareth. *The Blackwell companion to natural theology*, 593-662.

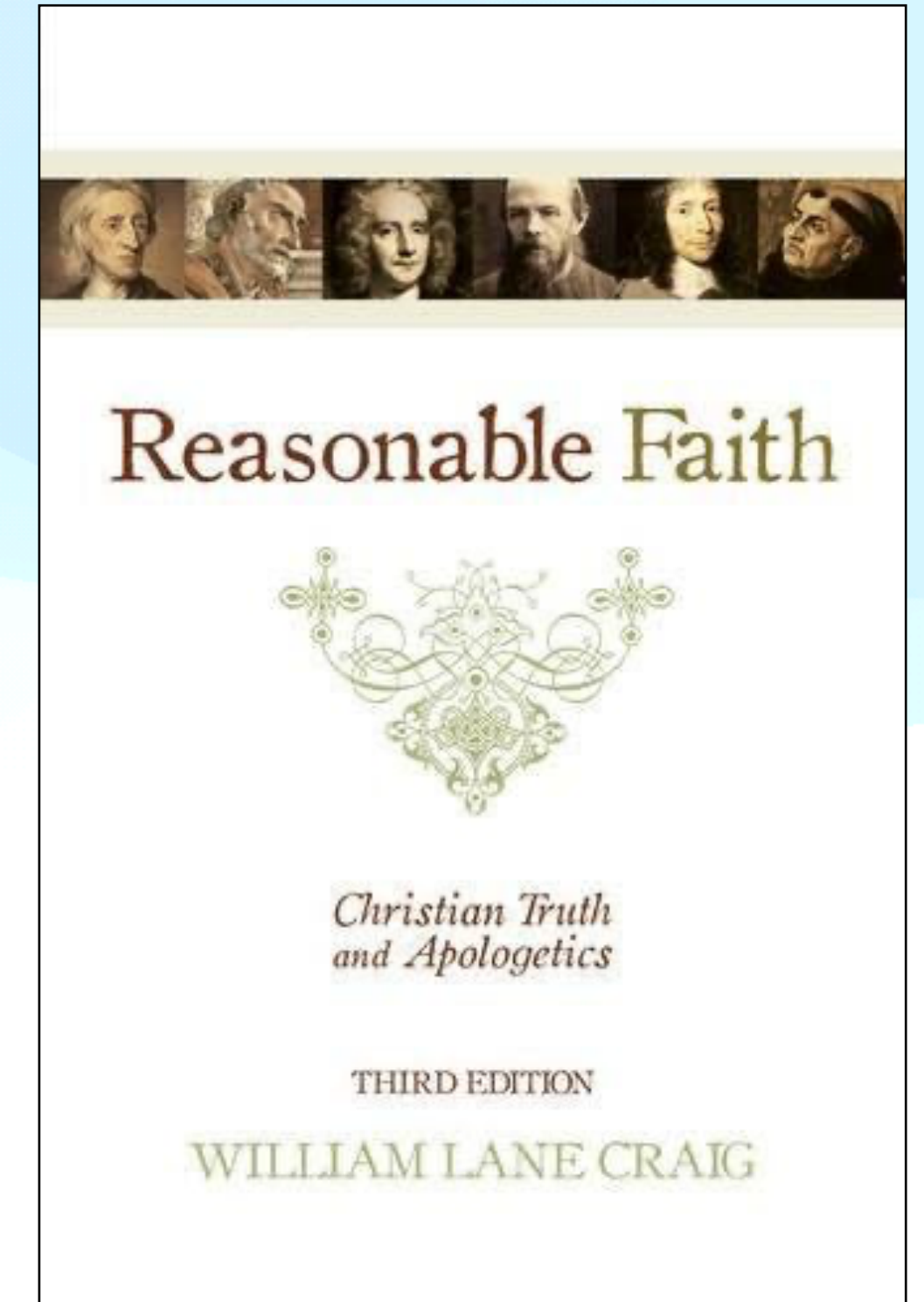
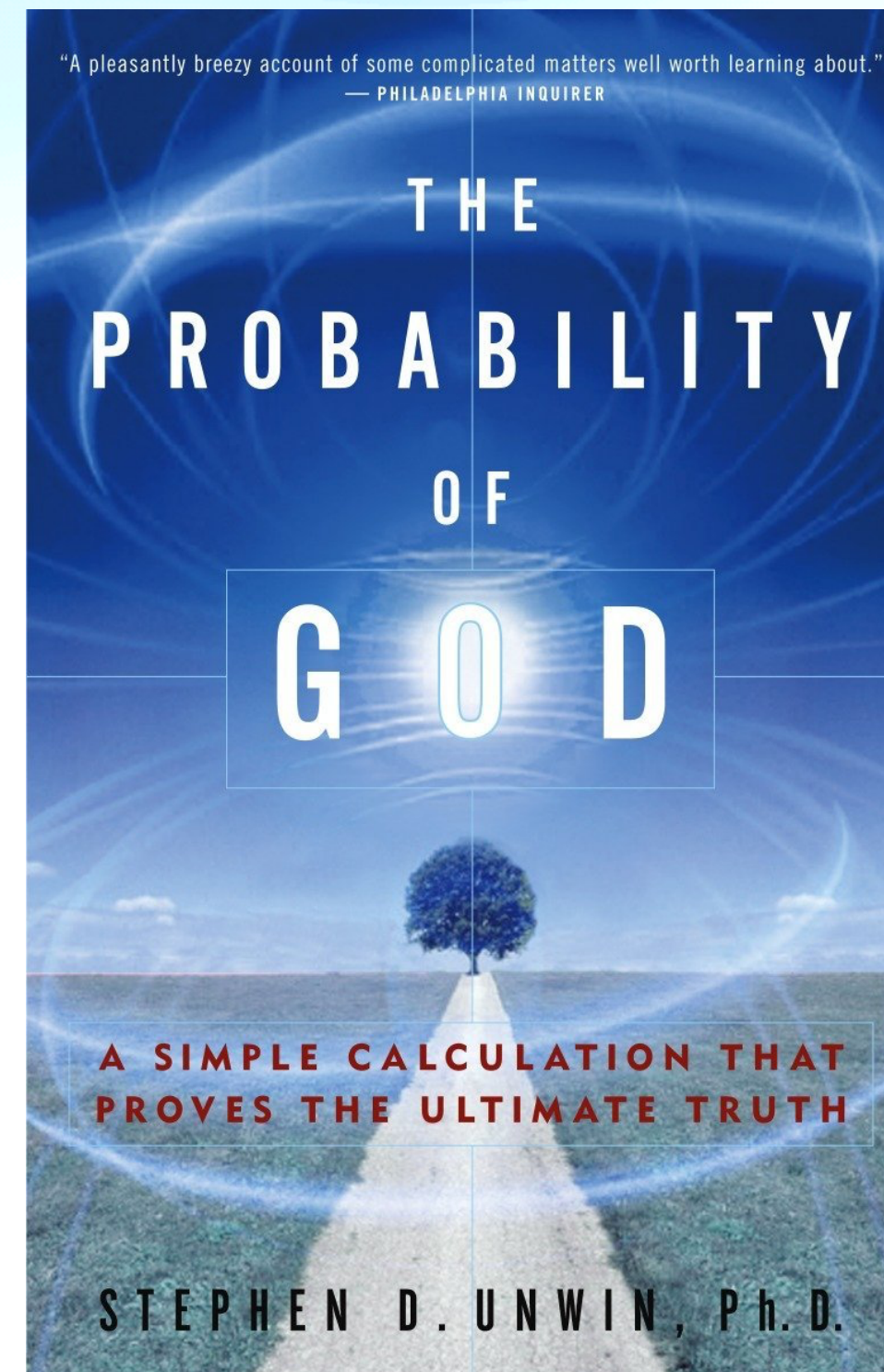
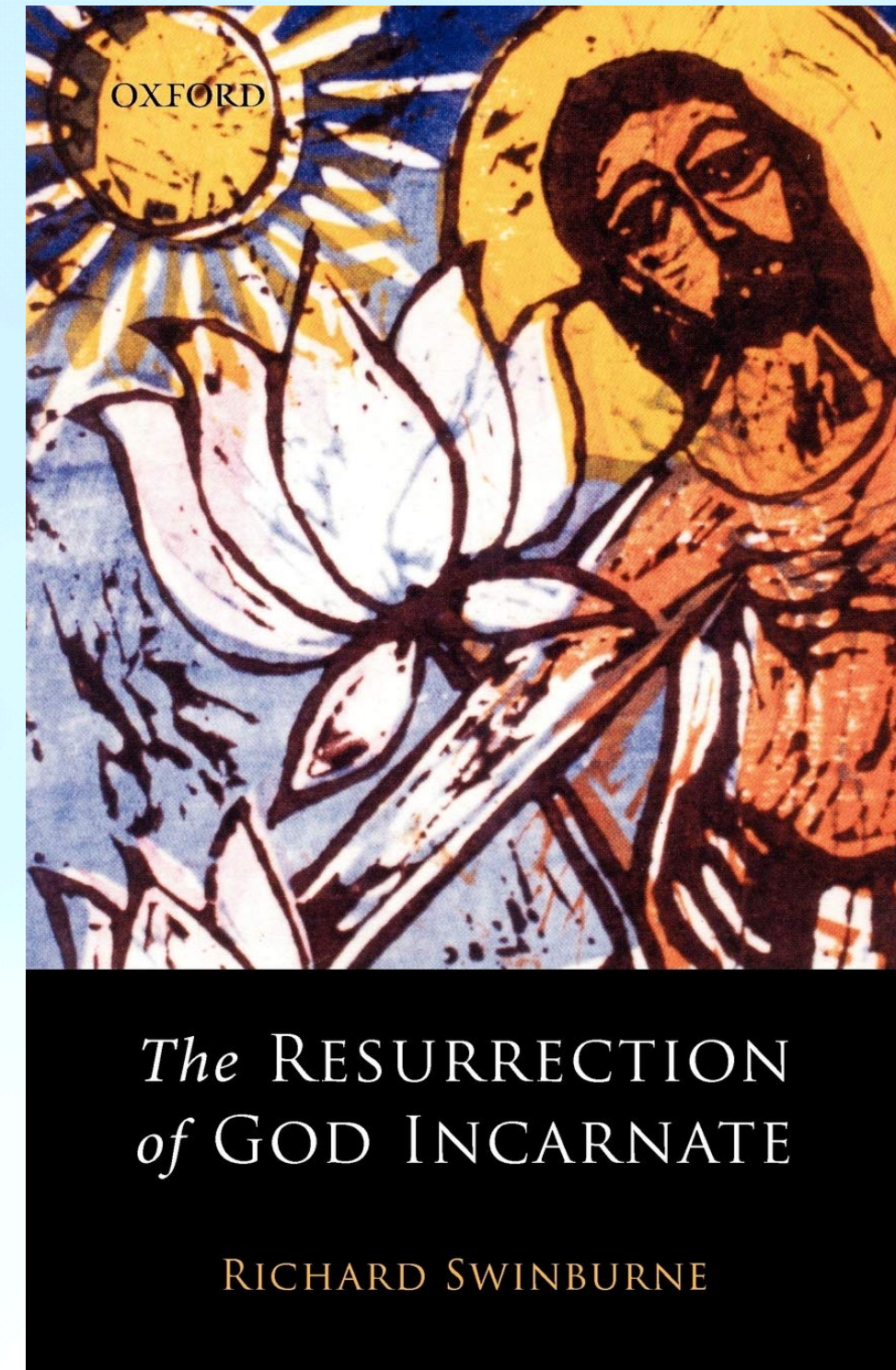
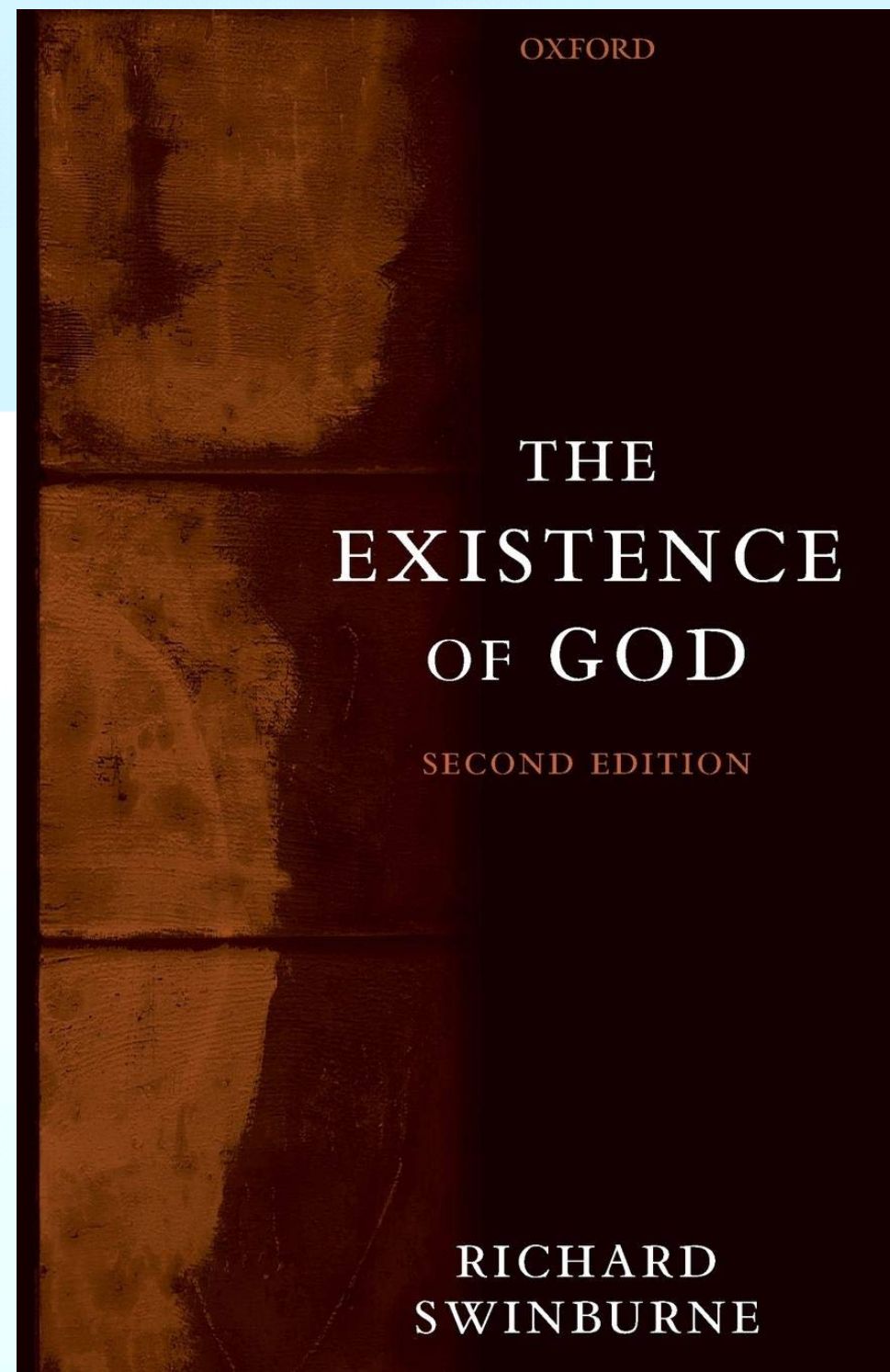
# The Game

## Mapping every concept in terms of probability

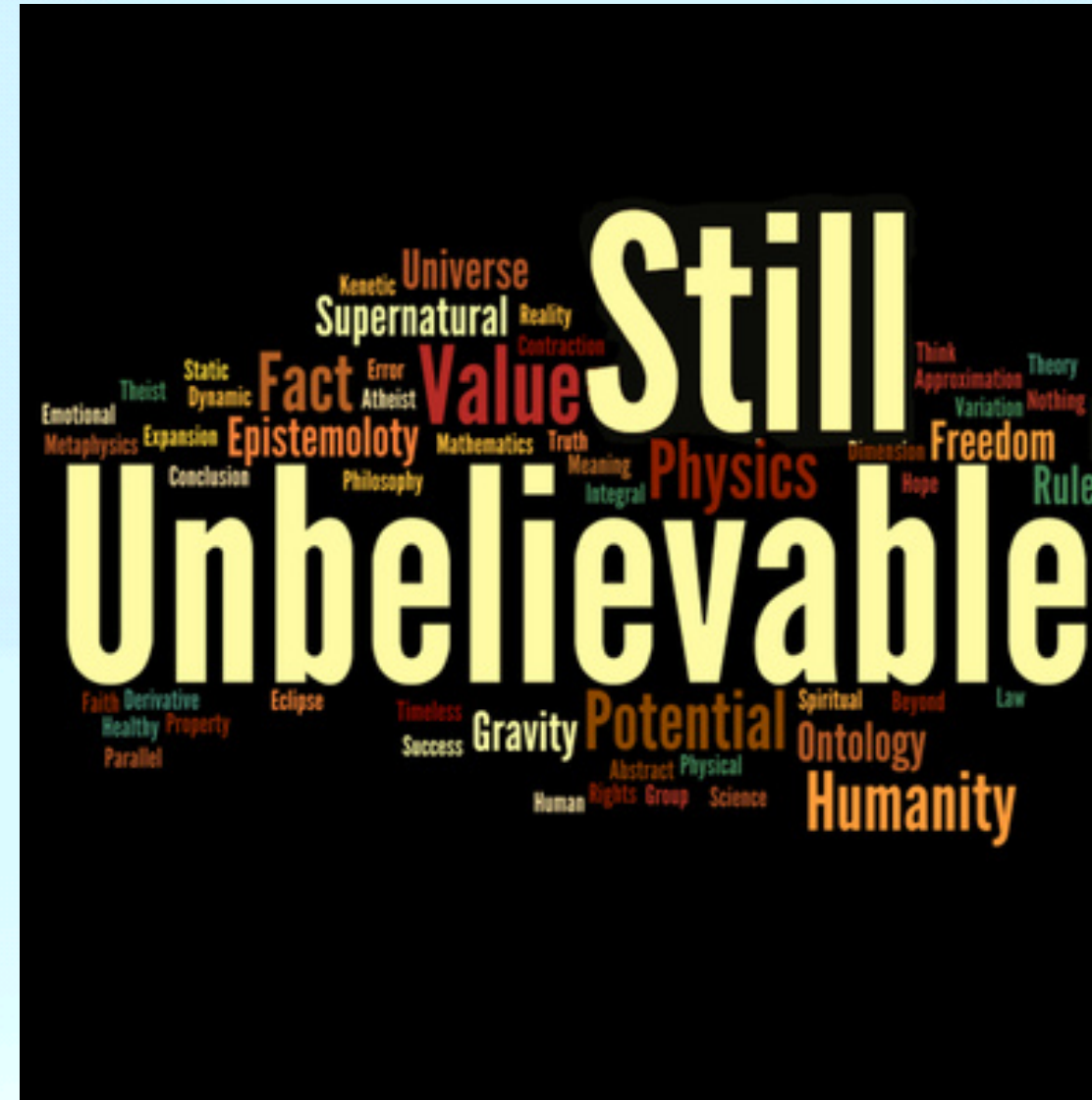
- Help structure the thought process
- Make explicit all your assumptions
- Uncover some unintuitive consequences
- Possibly make things less clear while appearing quantitative (hope not!)
- Possibly make things more clear with quantitative estimates



# Examples from Apologist Literature



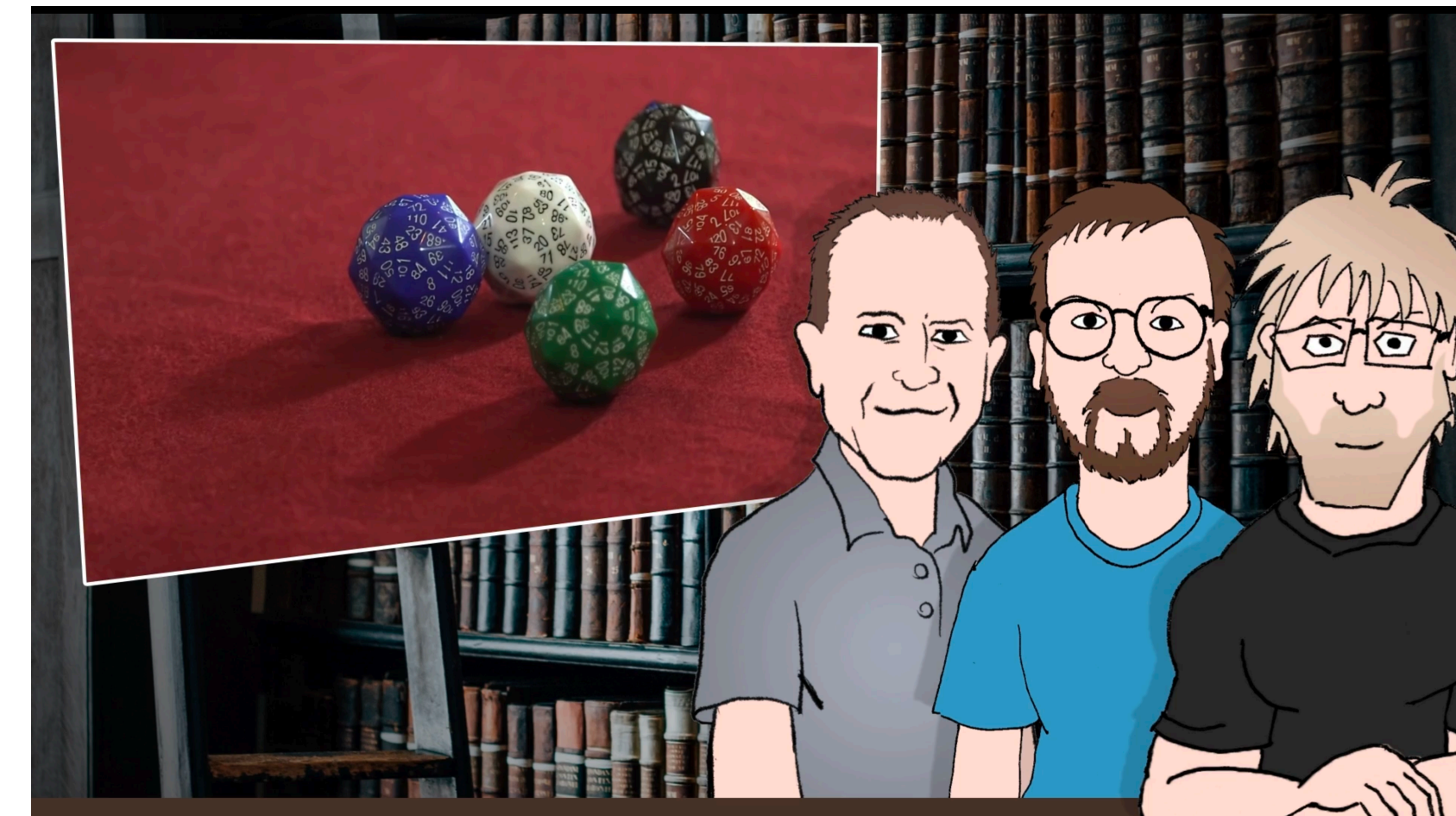
# Podcasts and YouTube



Digital Gnosis



Paulogia and MythVision



# Example Argument for God's Existence

## And its problems

*G: "there exists necessarily a person [mind] without a body (i.e. a spirit) who necessarily is eternal, perfectly free, omnipotent, omniscient, perfectly good, and the creator of all things" [clarification added]*

## data

1. the existence of a complex physical universe
2. the (almost invariable) conformity of material bodies to natural laws
3. those laws together with the initial state of the universe being such as to lead to the evolution of human organisms
4. these humans having a mental life (and so souls)
5. these humans having great opportunities for helping or hurting each other
6. these humans having experiences in which it seems to them that they are aware of the presence of God.

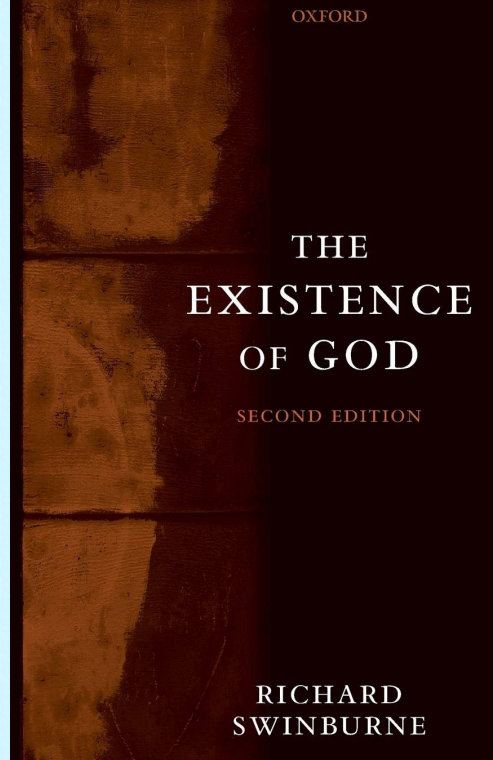
$$P(\text{data} | \sim G)P(\sim G) = P(\text{data} | H_1)P(H_1) + P(\text{data} | H_2)P(H_2) + P(\text{data} | H_3)P(H_3)$$

where

- $H_1$ : "there are many gods or limited gods"
- $H_2$ : "there is no God or gods but an initial (or everlasting) physical state of the universe, different from the present state but of such a kind as to bring about the present state"
- $H_3$ : "there is no explanation at all (the universe just is and always has been as it is)"

$$P(G | \text{data}) = \frac{P(\text{data} | G) \cdot P(G)}{P(\text{data} | G) \cdot P(G) + P(\text{data} | \neg G) \cdot P(\neg G)}$$

$$p(G | \text{data}) \sim 1/2$$



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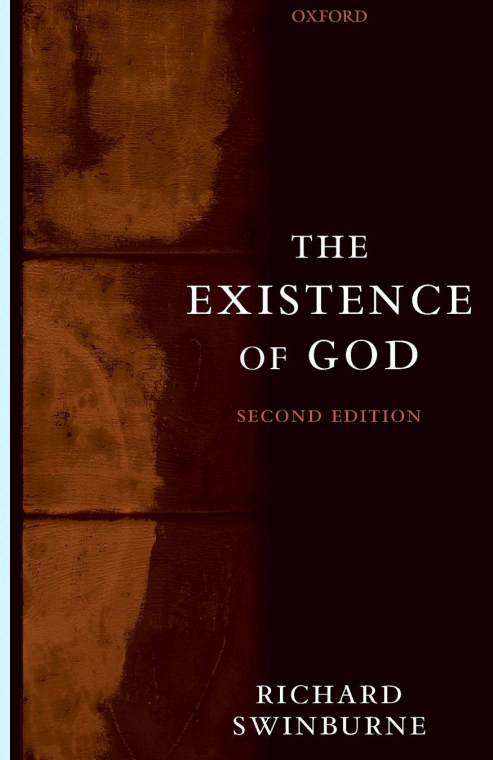
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$$p(G | \text{data}) \sim 1/2$$

The probabilities for these hypotheses are set quite low, given the reasons,

- $H_1$ : "hypothesis of theism is a very simple hypothesis indeed, simpler than hypotheses of many or limited gods"
- $H_2$ : "But there is no particular reason why an unextended physical point or any of the other possible starting points of the universe, or an everlasting extended universe, should as such have the power and liability to bring about all the features that I have described. [...] It will only become at all probable that there will be a universe of our kind if we build into the hypotheses an enormous amount of complexity. "
- $H_3$ : "And that our universe should have all the characteristics described (above all, the overwhelming fact that each particle of matter throughout vast volumes of space should behave in exactly the same way as every other particle codified in 'laws of nature') without there being some explanation of this is beyond belief. While  $P(\text{data} | H_3) = 1$  (the universe being this way unexplained entails it being this way),  $P(H_3)$  is infinitesimally low."



# Example Argument for God's Existence

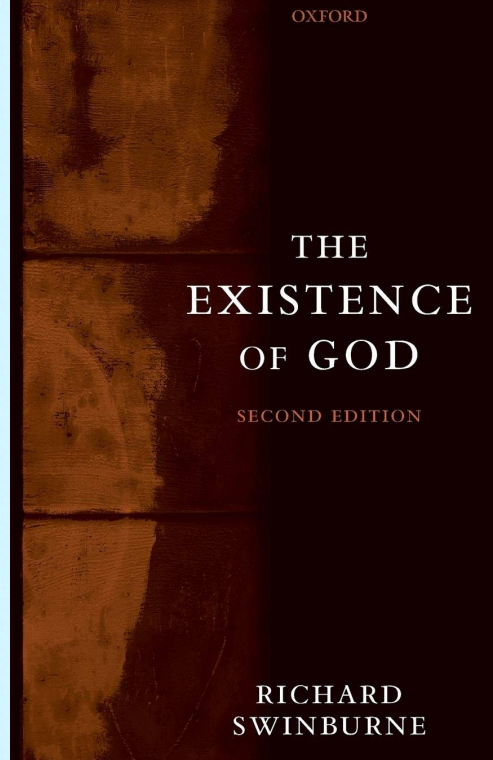
## And it's problems

- Lack of Imagination
- Ill-defined concepts
- Simplicity

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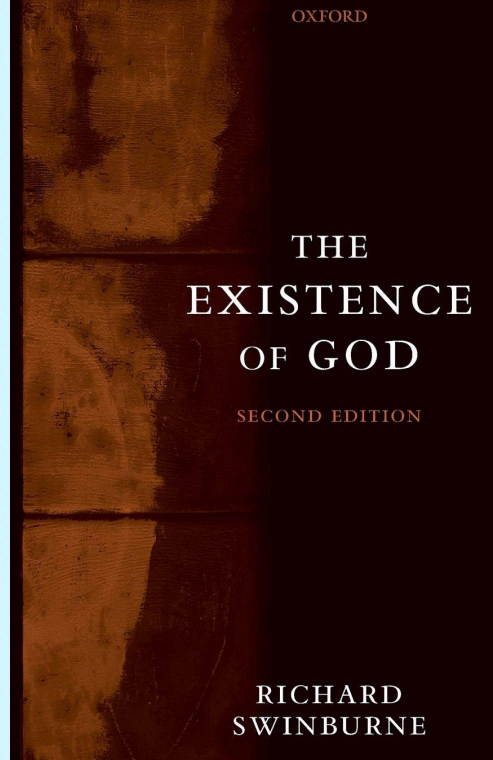
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# Lack of Imagination

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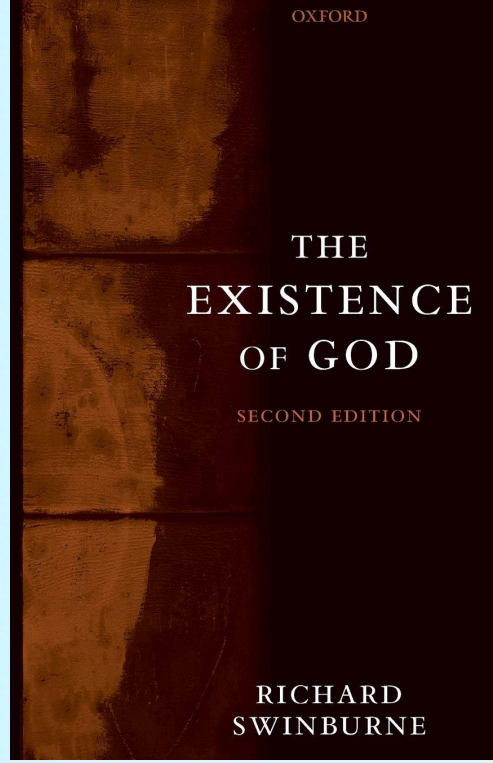
- $H_4$ : Stephen Law's Evil-God [[@law2010evil](#)]
- $H_5$ : Greek Pantheon, or any number of other mythos, exists. This is  $H_1$  broken up into specifics
- $H_6$ : Multiverse models (there could be many)
- $\vdots$
- etc...



# Simplicity

-  $H_1$ : "hypothesis of theism is a very simple hypothesis indeed, simpler than hypotheses of many or limited gods"

Apologists usually equate simplicity with the number of properties, or the length of the statement. Thus, a God explanation is simpler than one with Quantum Field Theory (QFT).

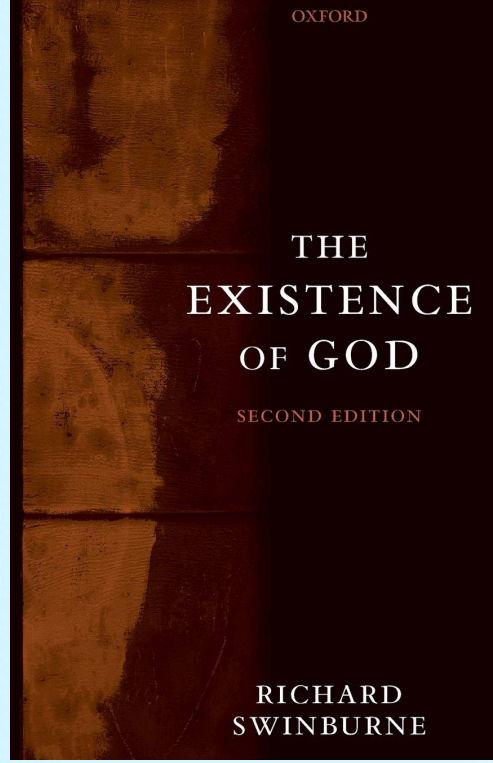


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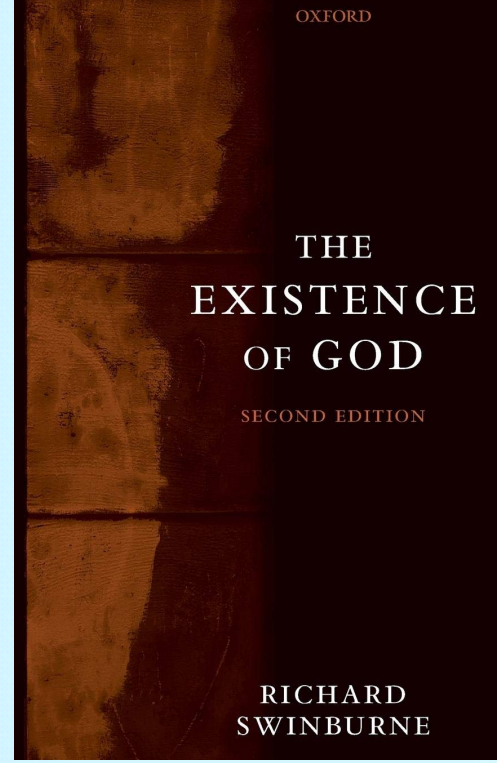
Probability theory states that simplicity concerns the flexibility of a model — more flexible models are more complex, and thus less likely. (Ockham Factor)





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$$P(M_1 | \text{data}) \sim P(\text{data} | M_1)P(M_1)$$

$$P(M_2 | \text{data}) \sim P(\text{data} | M_2)P(M_2)$$

$M_2$  has a parameter, call it  $\alpha$ , that can take on a range of values

$$P(M_1 | \text{data}) \sim P(\text{data} | M_1)P(M_1)$$

(marginalization  
results in a penalty  
for more complexity)

$$P(M_2 | \text{data}) \sim \int_{\alpha} P(\text{data} | M_2, \alpha)P(M_2 | \alpha)P(\alpha)$$

# Simplicity

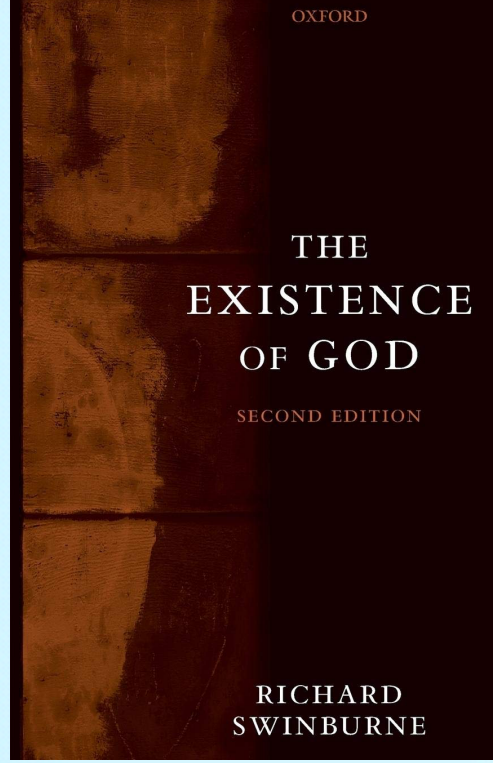
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Probability theory states that simplicity concerns the flexibility of a model — more flexible models are more complex, and thus less likely. (Ockham Factor)

Thus QFT (highly constrained) is simpler than the God explanation (which is unconstrained).

“Magic did it” would be just as unconstrained and thus equivalent in content and probability to “God did it”.



# Sagan's Maxim (often applied to miracle claims)

- Extraordinary claims require extraordinary evidence.
- Notation:  $M_0$  = extraordinary claim,  $M_1$  = all of the mundane claims

$$P(M_0 | \text{data}) \sim P(\text{data} | M_0)P(M_0)$$

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- If we assume the extraordinary claim fits the data perfectly,  $P(\text{data} | M_0) \approx 1$
- Then  $P(\text{data} | M_1) \ll 1$  or every other mundane claim must be nearly ruled out, not just unlikely

# Methods of Science

$$P(R | \text{data}) \sim P(\text{data} | R)P(R)$$

$$P(M | \text{data}) \sim P(\text{data} | M)P(M)$$

$$P(V | \text{data}) \sim P(\text{data} | V)P(V)$$

$$P(F | \text{data}) \sim P(\text{data} | F)P(F)$$

[...]

- Design experiments to rule out every other possible claim, otherwise the preferred model is made less probable by every other possible model with even a small probability

# Methods of Science

- Design experiments to rule out every other possible claim, otherwise the preferred model is made less probable by every other possible model with even a small probability
- Example: Measuring proton decay. Proton decay is one of the key predictions of the various grand unified theories (GUTs), is assumed to be absolutely stable in the Standard Model.
  - The Super-K is located 1,000 m (3,300 ft) underground
  - The 50 kilotons of pure water is continually reprocessed at rate about 30 tons/hour in a closed system
  - Removes dissolved gases in the water — these dissolved gases in water are a serious background event source
  - Membrane degasifier (MD) removes radon dissolved in water
  - ....

# Testimony and Independence

- David Hume: “No testimony is sufficient to establish a miracle unless it is of such a kind that its falsehood would be more miraculous than the fact that it tries to establish.”



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- For  $P(M_0 | \text{testimony}) > P(M_1 | \text{testimony})$  then  $\frac{P(\text{testimony} | M_1)P(M_1)}{P(\text{testimony} | M_0)P(M_0)} < 1$
- “miracle” simply means “low prior” so  $P(M_0) \ll 1; P(M_1) \approx 1$
- assume the miracle claim fits the data perfectly,  $P(\text{testimony} | M_0) \approx 1$
- Then  $P(\text{testimony} | M_1) \ll 1$  or the falsehood of the testimony (i.e. it would be true under the mundane explanation,  $M_1$ ) needs to be more improbable than the miracle itself,  $M_0$

# Testimony and Independence

- Timothy McGrew: “A sufficient number of independent testimonies, each of which has at least a certain minimum amount of force, will overcome any finite presumption against a miracle. Hume's "everlasting check" fails; a cumulative case can, in principle, make any miracle claim credible.”  
(Emphasis mine)

$$P(R | D_n) \sim P(D_n | R)P(R)$$

$$P(\neg R | D_n) \sim P(D_n | \neg R)P(\neg R)$$

$$\left( \frac{P(D_1 | R)}{P(D_1 | \neg R)} \right)^n \frac{P(R)}{P(\neg R)} = \left( \frac{1000}{1} \right)^n \frac{P(R)}{P(\neg R)}$$

# Testimony and Independence

- Some notation  $P(R) \equiv m, P(\neg R) \equiv 1 - m$
- Several data points  $\{D_i\} \equiv D_1, D_2, D_3, \dots, D_N$
- Likelihood for each data point the same  $P(D_i | R) = d, P(D_i | \neg R) = b$ 
  - McGrews use  $d/b = 1000$
- Fully independent solution  $P(D_i | \{D_1, \dots, D_{i-1}\}, R) = P(D_i | R)$

$$\frac{P(R | D_1, D_2, D_3, \dots, D_N)}{P(\neg R | D_1, D_2, D_3, \dots, D_N)} = \left(\frac{d}{b}\right)^N \cdot \frac{m}{1 - m}$$

Because

$$P(\{D_i\} | R) = P(D_1 | R) \cdot P(D_2 | D_1, R) \cdot P(D_3 | D_1, D_2, R) \cdots P(D_N | D_1, D_2, \dots, D_{N-1}, R)$$

# Testimony and Independence

- Fully independent solution  $P(D_i | \{D_1, \dots, D_{i-1}\}, R) = P(D_i | R)$

$$\frac{P(R | D_m)}{P(\neg R | D_m)} = \left(\frac{d}{b}\right)^N \cdot \frac{m}{1 - m}$$

- Fully dependent solution  $P(D_i | \{D_1, \dots, D_{i-1}\}, R) = 1$  for  $i \neq 1$

$$\frac{P(R | D_m)}{P(\neg R | D_m)} = \left(\frac{d}{b}\right) \cdot \frac{m}{1 - m}$$

Because

$$P(\{D_i\} | R) = P(D_1 | R) \cdot P(D_2 | D_1, R) \cdot P(D_3 | D_1, D_2, R) \cdots P(D_N | D_1, D_2, \dots, D_{N-1}, R)$$

# Model of the Uncertainty in the Independence of Testimony

- $\beta = 1$  we're certain the data point  $D_2$  is independent of  $D_1$ :  $P(D_2 | R, D_1) = d$
- $\beta = 0$  we're certain the data point  $D_2$  is dependent of  $D_1$ :  $P(D_2 | R, D_1) = 1$

$$\begin{aligned} O &\equiv \frac{P(R | D_1, D_2, \dots, D_N)}{P(\neg R | D_1, D_2, \dots, D_N)} \\ &= \frac{(\beta d + (1 - \beta))^{N-1} \cdot d}{(\beta b + (1 - \beta))^{N-1} \cdot b} \cdot \frac{P(R)}{P(\neg R)} \end{aligned}$$

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- Reproduce the McGrew calculation
  - $\beta = 1$  we're certain of independence,  $d = 10^{-3}$ ,  $b = 10^{-6}$ ,  $N = 15$

$$O = 10^{45} \frac{P(R)}{P(\neg R)}$$

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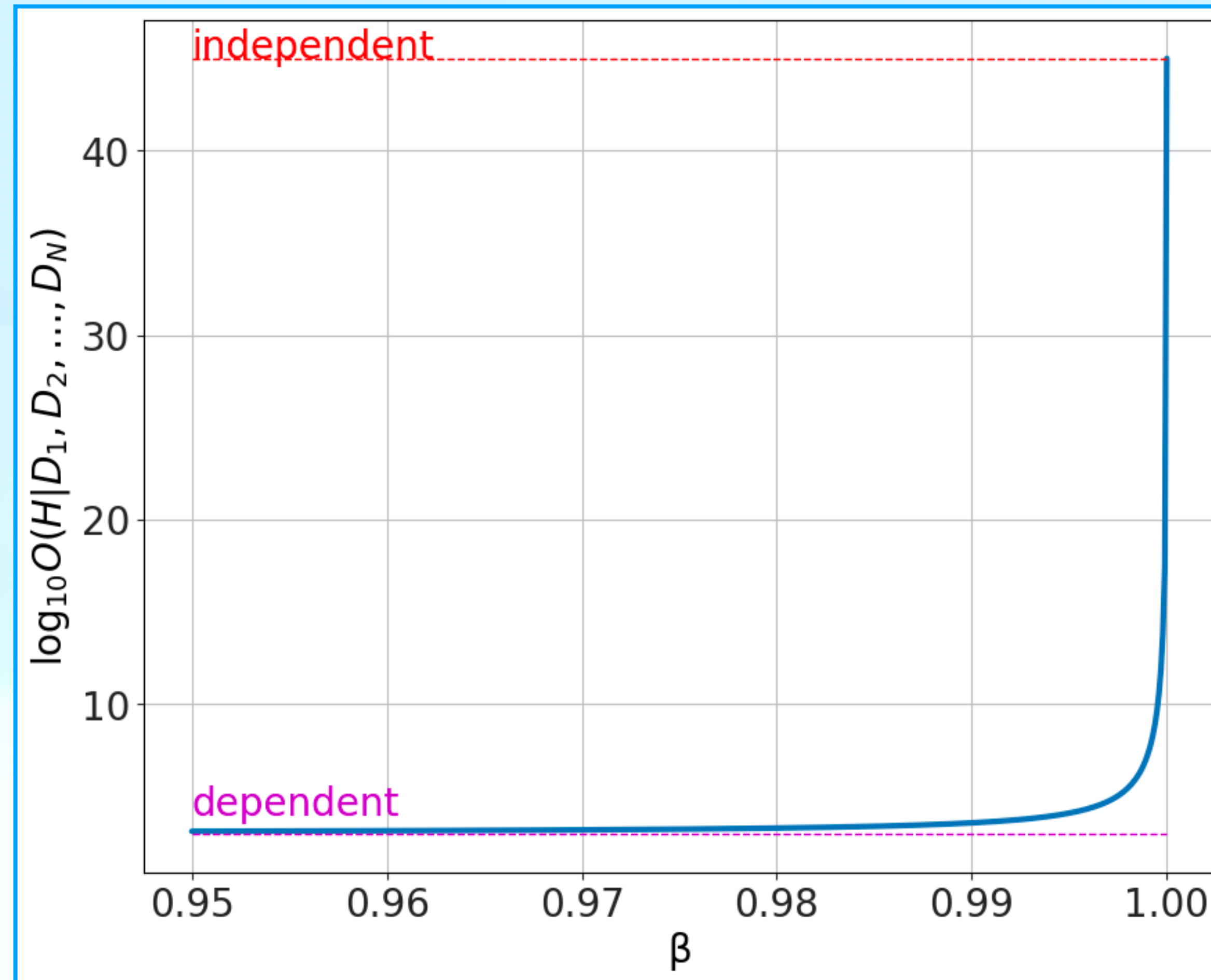
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- Reproduce the McGrew calculation
  - $\beta = 1$  we're certain of independence,  $d = 10^{-3}$ ,  $b = 10^{-6}$ ,  $N = 15$

How certain are we of independence?

$$O = 10^{45} \frac{P(R)}{P(\neg R)}$$

# Model of the Uncertainty in the Independence of Testimony



The tiniest deviation from the absolute certainty that all 15 sources are statistically independent brings the odds ratio down to the mundane!

One can be supremely confident that all 15 sources are statistically independent, at probability of  $p = 0.9995$  (which is far higher than many scientific claims in published journals), and still not be able to justify the miracle claim due to the small uncertainty.



# Investigating Miracle Claims and Probability

- I've personally looked into many miracle claims, and pseudoscience claims (e.g. UFO sightings, alien abductions, magnetic therapy, etc...) They've all failed mostly for mundane reasons
  - Data not available
  - No proper timeline for effect
  - Obvious mundane explanations
  - No controlled observations
  - Unreliable witnesses

# Investigating Miracle Claims and Probability

- A model inspired by an example in E. T. Jaynes
- We have the proposition,  $M \equiv$  a miracle occurred, for which we have a prior,

$$P(M) \equiv m$$

$$P(\bar{M}) \equiv 1 - m$$

- Our data consists not of extraordinary observations of the world but of a series of claims about such observations. This can include sources such as
  - statements from people who made the observations
  - texts, in this case ancient texts, which include the claims
  - second- and third-hand accounts of observations
- For the sake of concreteness, I'll say that the data is  $C \equiv$  person X has made a claim of  $M$ .

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- The data is  $C \equiv$  person  $X$  has made a claim of  $M$
- For the sake of charity, we will assume that if a miracle has occurred, then the person would make that claim with certainty,  $P(C | M) = 1$ . (Also for charity, we will ignore data that we'd expect under  $M$  but do not observe).
- Someone may make a claim of a miracle even if its negation,  $\neg M$ , is actually true. I simplify this to some constant probability,  $P(C | \neg M) = a$

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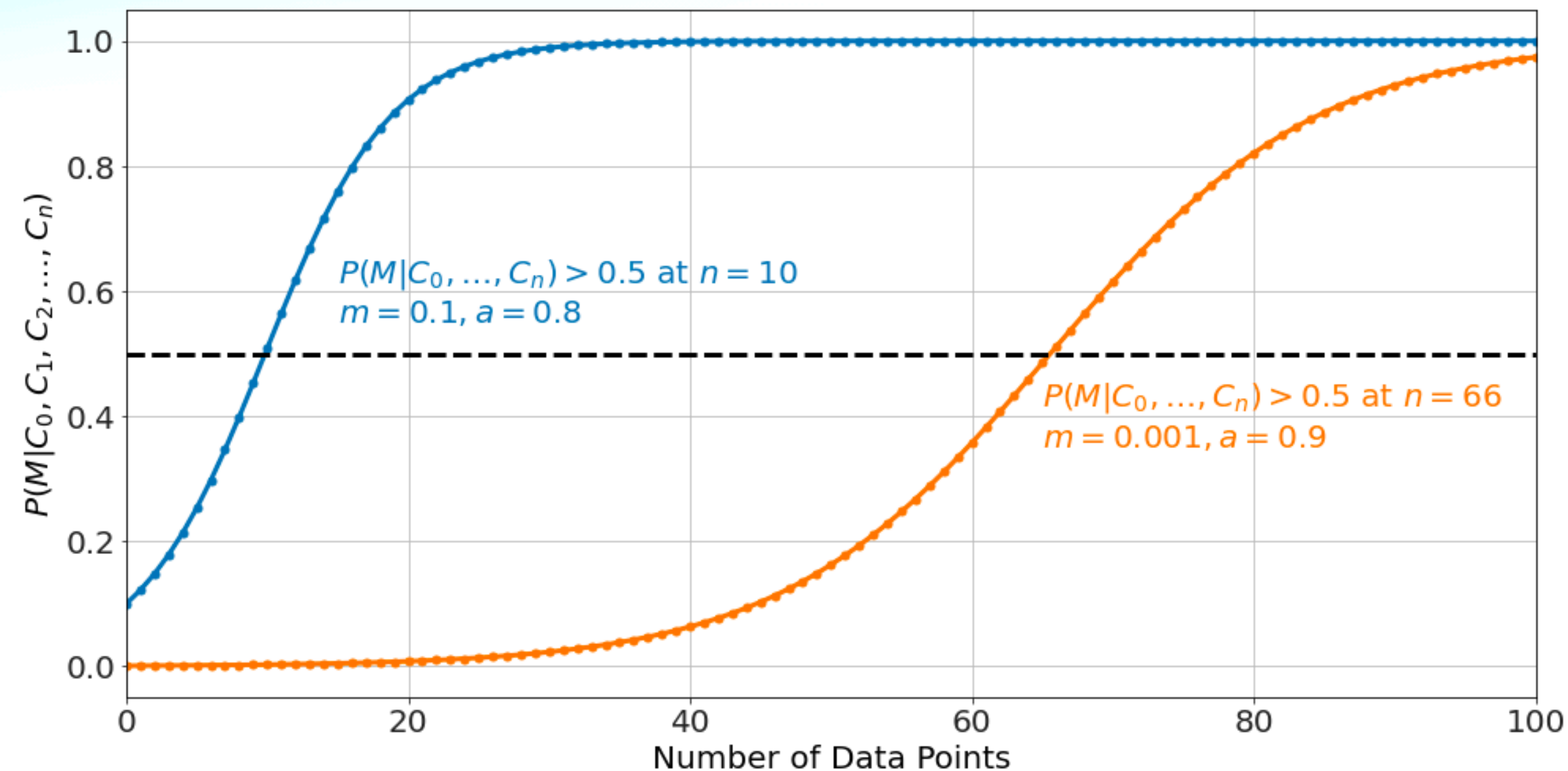
$$P(C | M) = 1$$

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We can think of  $a$  as a measure of the reliability (or rather unreliability) of the source, with  $a = 0$  being completely reliable and  $a = 1$  being (nearly) unreliable

$$P(M | C_0, C_1, C_2, \dots, C_n) = \frac{m}{m + a^n \cdot (1 - m)}$$

This part is nothing new — it's just the same as the multiple independent sources overcoming any prior result from before



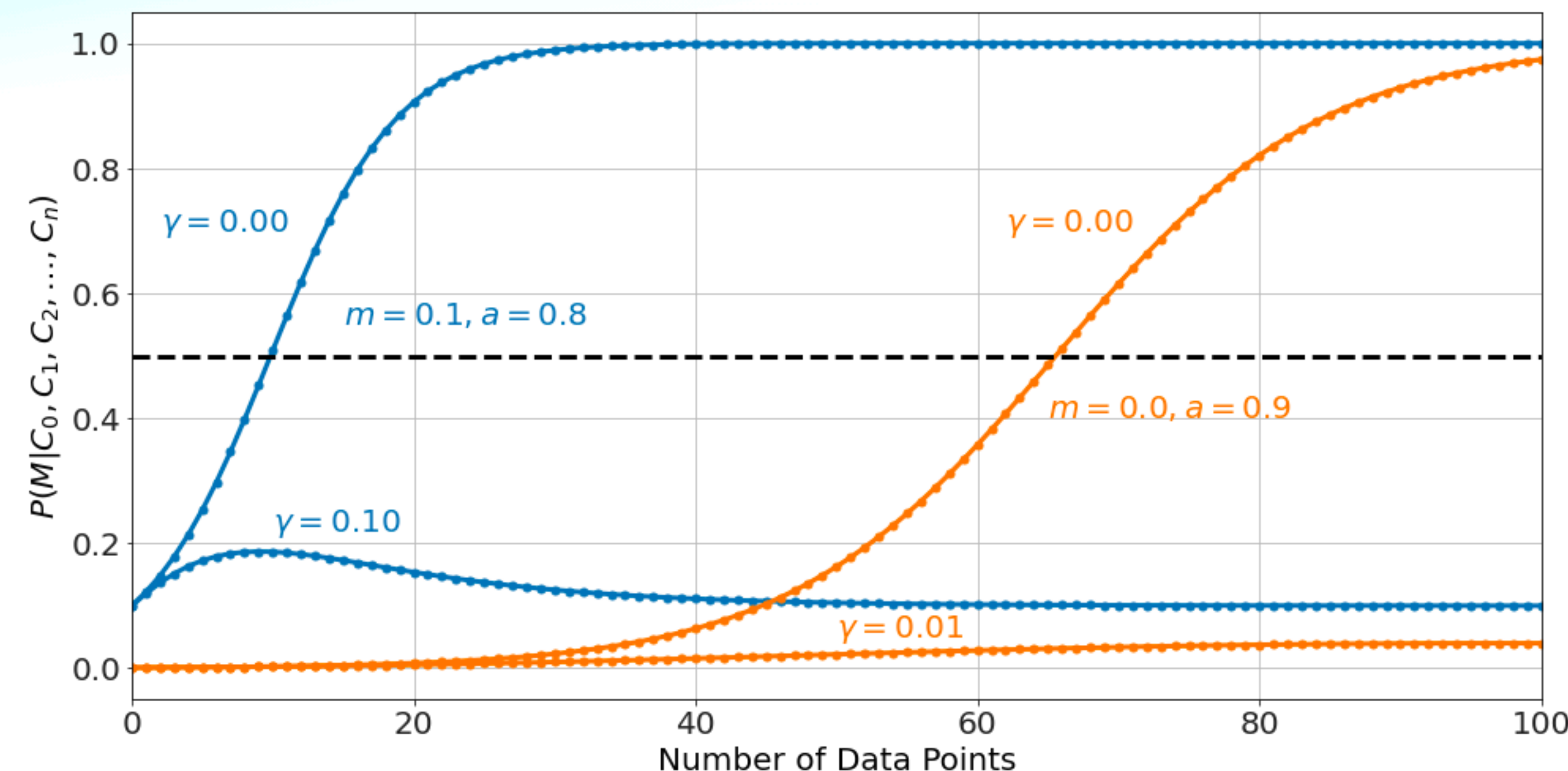
# Investigating Miracle Claims and Probability

- Introduce science. What I've observed is: Claim 1 → debunking of Claim 1 → Claim 2 → debunking of Claim 2 → Claim 3 → debunking of Claim 3 → ...
- Each debunking makes the next claim less reliable:  $a \rightarrow a + \gamma \cdot (1 - a)$

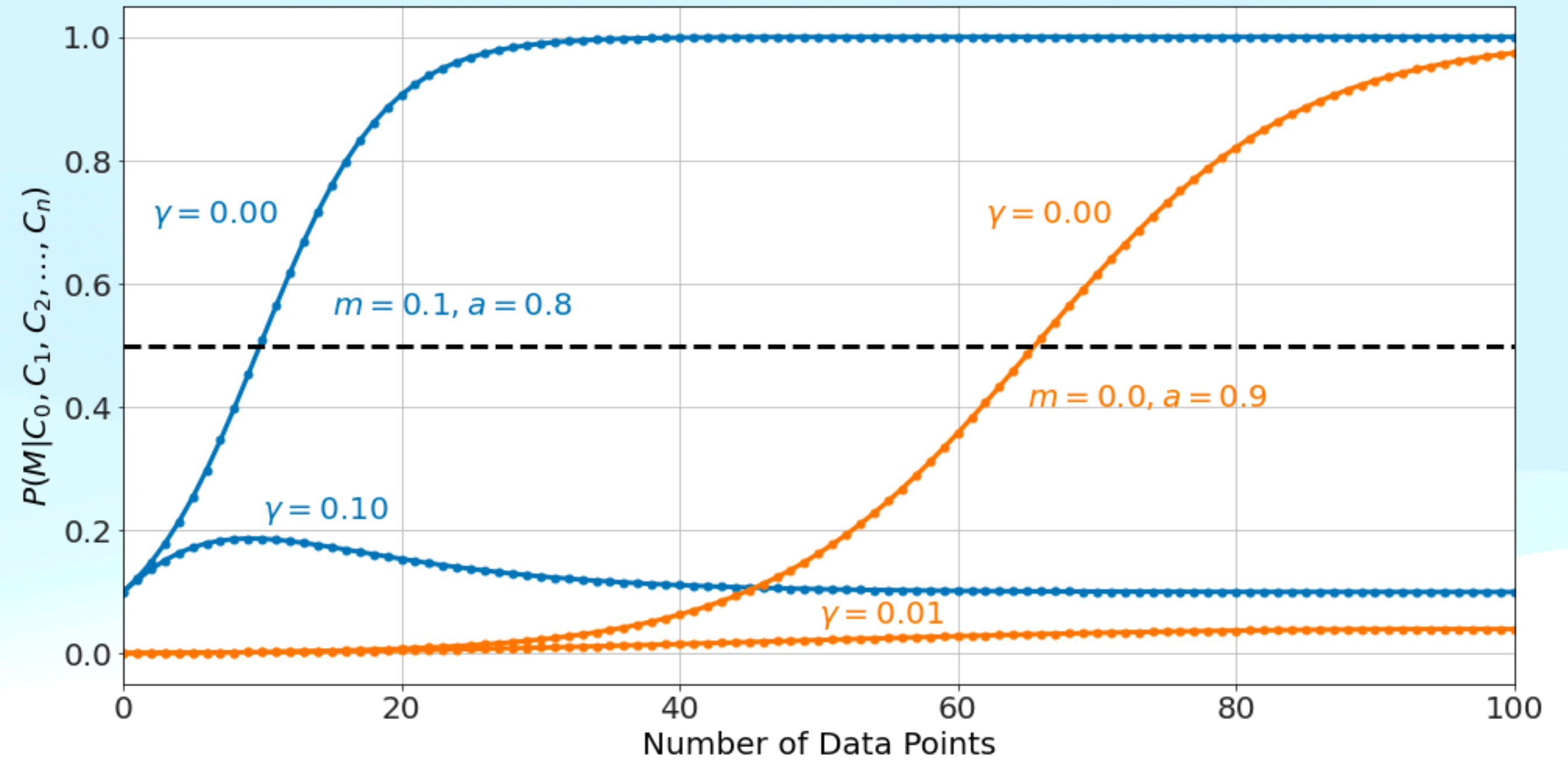
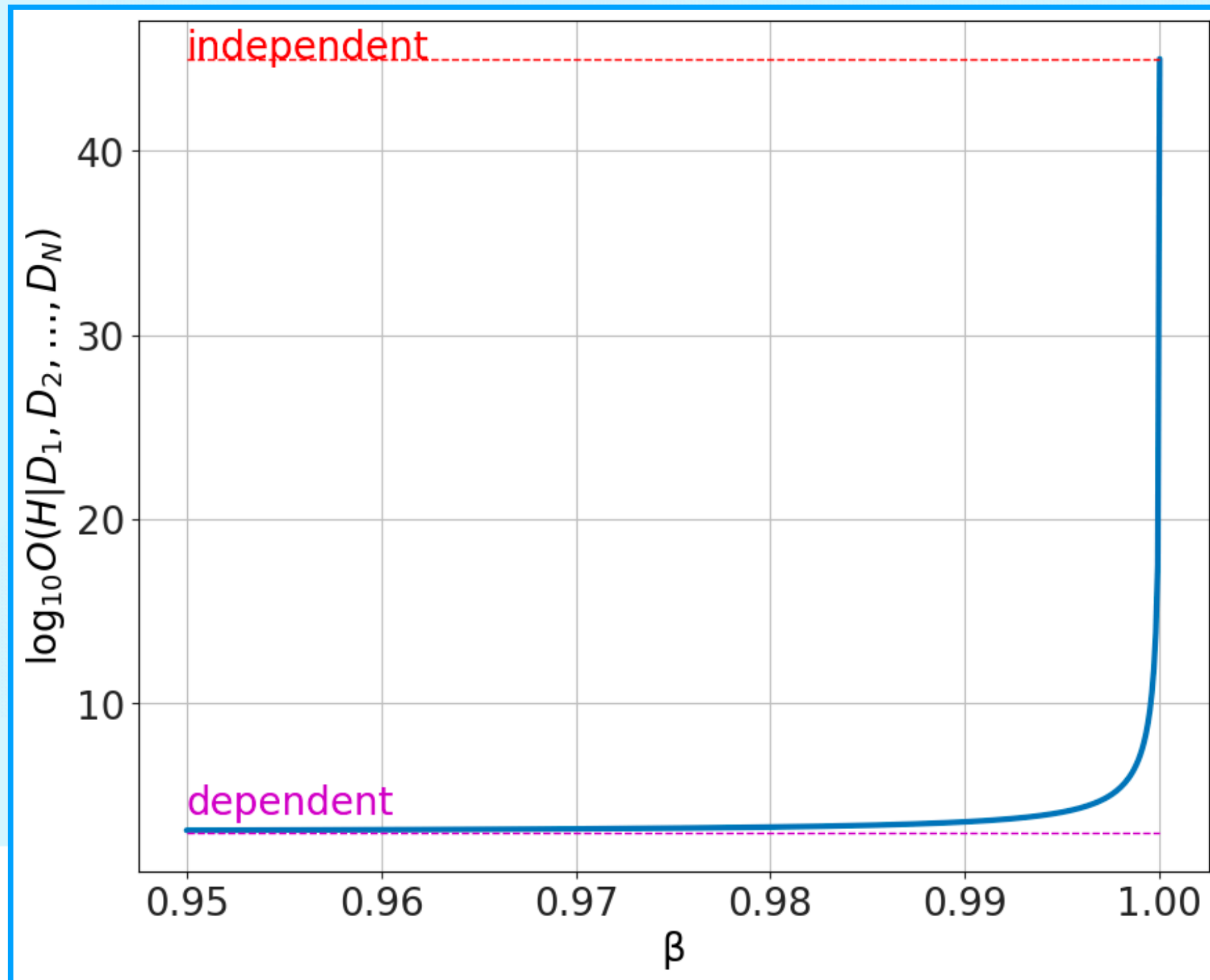
$$P(M | C_0, C_1, C_2, \dots, C_n) = \frac{m}{m + ((a_0 + \gamma - 1) \cdot (1 - \gamma)^{n-1} + 1)^n \cdot (1 - m)}$$

Further testimony reduces the probability of claims back to the prior.

We can think of  $a$  as a measure of the reliability (or rather unreliability) of the source, with  $a = 0$  being completely reliable and  $a = 1$  being (nearly) unreliable



# Testimony



**Dr. Jonathan McLatchie** @JMcLatchie\_ · Jun 13

Do you think testimony provides no evidence that a miracle has occurred, or do you merely think it provides insufficient evidence? If the latter, would you agree that in principle the testimony could be of sufficient quantity and quality that it is sufficient?

5



4



# Faith and Trust

- What is Faith?
  - “Belief without [sufficient] evidence” - Peter Boghossian, Matt Dillahunty
  - “Faith is trusting in, holding to, and acting on what one has good reason to believe is true in the face of difficulties. The difficulties may be where you have to take an action where the outcome is beyond your control.” - Tim McGrew
- Example: jumping out of an airplane

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- Example: jumping out of an airplane
- Understood as expected utility: Faith =  $\sum_i \underbrace{P(C_i)}_{\text{belief}} \cdot \underbrace{U(C_i)}_{\text{action}}$
- I personally find people use the word in both cases, and probably should be avoided for clarity — just use words like belief, evidence, and trust



# Apologists like to ignore priors

<https://jonathanmclatchie.com/bayesian-probability-and-the-resurrection-a-reply-to-brian-blais/>

“It is not necessary for a hypothesis to be able to make high probability predictions in order for it to be well evidentially supported. Rather, it is only necessary that the pertinent data be rendered more probable given the hypothesis than it would be on its falsehood. Suppose you are walking in a forest and stumble upon a shack that, upon initial inspection, appears to be uninhabited. Nonetheless, you decide to investigate. As you open the door, you notice a table, upon which there is a tumbler containing Earl Grey tea, which is still steeping. Now, on the hypothesis that the shack is inhabited, does it predict with high probability the presence of the steeping Earl Grey tea on the table? Hardly! Nonetheless, this observation is very strong evidence that the shack is inhabited, since on that supposition the presence of the tea (even though improbable) is far, far more probable than it would be on the falsehood of that hypothesis. **What is important, then, is the likelihood ratio of the probabilities.**”

# Apologists like to ignore priors

<https://jonathanmclatchie.com/bayesian-probability-and-the-resurrection-a-reply-to-brian-blais/>

“It is not necessary for a hypothesis to be able to make high probability predictions in order for it to be well evidentially supported. Rather, it is only necessary that the pertinent data be rendered more probable given the hypothesis than it would be on its falsehood. Suppose you are walking in a forest and stumble upon a shack that, upon initial inspection, appears to be uninhabited. Nonetheless, you decide to investigate. As you open the door, you notice a table, upon which there is a tumbler containing Earl Grey tea, which is still steeping. Now, on the hypothesis that the shack is inhabited, does it predict with high probability the presence of the steeping Earl Grey tea on the table? Hardly! Nonetheless, this observation is very strong evidence that the shack is inhabited, since on that supposition the presence of the tea (even though improbable) is far, far more probable than it would be on the falsehood of that hypothesis. **What is important, then, is the likelihood ratio of the probabilities.**”

- two models —

Priors

$$P(H) = 1/10$$

$$P(U) = 9/10$$

- inhabited ( $H$ )

Likelihoods

$$P(\text{hot tea} | H) \sim 1/1000$$

$$P(\text{hot tea} | U) \sim 1/1,000,000$$

- uninhabited ( $U$ )

Posteriors

$$P(H | \text{hot tea}) \sim P(\text{hot tea} | H) \times 1/10 = 92 \%$$

$$P(U | \text{hot tea}) \sim P(\text{hot tea} | U) \times 9/10 = 8 \%$$

# Apologists like to ignore priors

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“It is not necessary for a hypothesis to be able to make high probability predictions in order for it to be well evidentially supported. Rather, it is only necessary that the pertinent data be rendered more probable given the hypothesis than it would be on its falsehood. Suppose you are walking in a forest and stumble upon a shack that, upon initial inspection, appears to be uninhabited. Nonetheless, you decide to investigate. As you open the door, you notice a table, upon which there is a tumbler containing Earl Grey tea, which is still steeping. Now, on the hypothesis that the shack is inhabited, does it predict with high probability the presence of the steeping Earl Grey tea on the table? Hardly! Nonetheless, this observation is very strong evidence that the shack is inhabited, since on that supposition the presence of the tea (even though improbable) is far, far more probable than it would be on the falsehood of that hypothesis. **What is important, then, is the likelihood ratio of the probabilities.**”

- two models —

Priors

$$P(H) = 1/10$$

$$P(J) = 1/1,000,000,000$$

$$P(U) = 9/10$$

- inhabited ( $H$ )

- uninhabited ( $U$ )

Likelihoods

$$P(\text{hot tea} | H) \sim 1/1000$$

$$P(\text{hot tea} | J) \sim 1$$

- Jean Luc Picard ( $J$ )

$$P(\text{hot tea} | U) \sim 1/1,000,00$$

Posteriors

$$P(H | \text{hot tea}) \sim P(\text{hot tea} | H) \times 1/10 = 92 \%$$

$$P(J | \text{hot tea}) \sim 1/10,000 \%$$

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# Conclusions

## The Game

### Mapping every concept in terms of probability

- Help structure the thought process
- Make explicit all your assumptions
- Uncover some unintuitive consequences
- Possibly make things less clear while appearing quantitative
- Possibly make things more clear with quantitative estimates

- Lessons:
  - $P(H|data)$  depends on the alternatives considered even more than the data sometimes
  - What we call “data” has to be carefully considered
  - Even a model with a very low prior can become really likely if the event being described is rare
  - Statistical independence has to be demonstrated and may not always apply
  - Models have to be well-defined
  - You can introduce data which is more likely on a given model and have the probability of that model go down

<https://bblais.github.io>

@bblais on X (Twitter)

# What Does Probability have to do with God?

Exploring Bayesian Reasoning in Theological Problems

*Thanks!*

secular

society

of MIT

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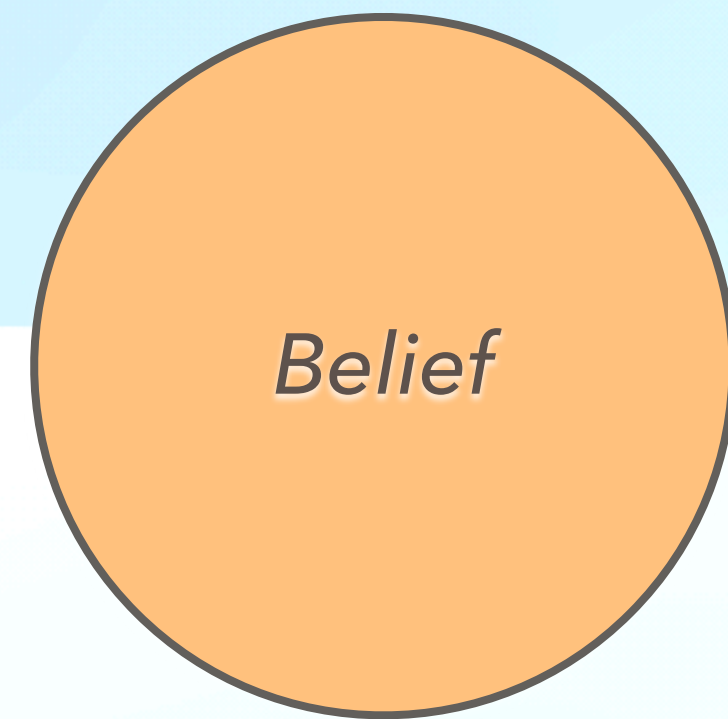


**Extra**

# Basic Definitions of Philosophical Terms

- Worldview
- Atheism
- Anti-theism
- Not a fan of the labels — if they don't work for you, then don't use them
- Worldview and priors <https://bblais.github.io/posts/2016/Sep/06/mapping-worldview-to-probability/>
- <https://bblais.github.io/posts/2021/Feb/28/bayes-vs-apologetics/>
- <https://bblais.github.io/posts/2014/Jan/24/properly-basic-obscurantism/>
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# Process of Science



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*How well does my explanation explain the data?*

x

*How plausible is my explanation before data?*

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*All of the other ways this data could be explained*



# Basic Definitions of Philosophical Terms

*Claim:  $S :=$  There are an even number of stars in the Galaxy.*

- Believer - I believe the claim is true (i.e. there are an even number of stars)
- Non-Believer - I don't believe the claim — can you give evidence for it?
- Anti-believer - I believe the claim is false (i.e. there are an odd number of stars)
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# Basic Definitions of Philosophical Terms

*Claim:  $S :=$  There are an even number of stars in the Galaxy.*

- Believer - I believe the claim is true (i.e. there are an even number of stars)
  - $P(S) > 0.9$
- Non-Believer - I don't believe the claim — can you give evidence for it?
  - $P(S) \sim 0.5$
- Anti-believer - I believe the claim is false (i.e. there are an odd number of stars)
  - $P(S) < 0.1$
- Not a fan of the labels — if they don't work for you, then don't use them

# Basic Definitions of Philosophical Terms

*Claim G: There is a God.*

- Theist - I believe the claim is true (i.e. there is a God)
  - $P(G) > 0.9$
- Atheism - I don't believe the claim — can you give evidence for it?
  - $P(G) \sim 0.5$  or  $P(G) \sim \text{Uniform}(0,1)$  or  $G$  is not well defined
- Anti-theism - I believe the claim is false (i.e. there is not a God)
  - $P(G) < 0.1$
- Note: not a fan of the labels — if they don't work for you, then don't use them